

# Cultural Goods Consumption and Cultural Capital<sup>1</sup>

## 1. Introduction

It is a widespread presumption and/or conviction that the consumption of cultural goods yields positive effects<sup>2</sup> for society: the creativity of mankind will be further developed, the tolerance for others (race and gender) will be enhanced and even the criminality will be reduced etc. In economic terminology, the consumption of culture involves a positive externality. Through the repeated consumption of cultural goods cultural capital<sup>3</sup> will be accumulated over time and that cultural capital, in turn, is appreciated by all members of society. Accordingly, the more cultural goods are consumed, the more cultural capital is generated and the greater will be the external benefits provided for society.

Though in the laissez-faire economy consumers realize the positive effects generated by the consumption of cultural goods, they tend to ignore the beneficial impact their own contributions to the generation of cultural capital has on their fellow citizens, and, when the number of consumers is large, they may even ignore that their own utility is enhanced through an increase in cultural capital induced by their own consumption of cultural goods. As a result, cultural capital cannot be generated efficiently in the laissez-faire economy. Since there is no market for cultural capital, it will be underprovided.

The external benefits of cultural goods consumption are quite well understood and the external benefits argument for government subsidy in static equilibrium is extensively discussed by e.g. Netzer (1978), Fullerton (1992), Sawers (1993). Peacock (1969) considers the welfare of future generations as an argument for public subsidies to the art. Scandizzo (1993) studies the cultural consumption behavior in dynamic analysis.

This paper provides a justification for subsidies on cultural goods that serve to internalize the external benefits of cultural goods consumption. Although the basic argument has a strong

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<sup>1</sup> Helpful comments by Thomas Eichner, Marco Runkel, Andreas Wagener are gratefully acknowledged. Remaining errors are the authors' responsibility.

<sup>2</sup> A large literature discusses the effects, see e.g. The First World Culture Report (2000) of UNESCO.

<sup>3</sup> On the terminology of cultural capital see also David Throsby (1999).

common sense intuition, to our knowledge it has not yet been developed and demonstrated in the framework of a dynamic theoretical model.

Positive externalities of cultural goods consumption are introduced in a dynamic setting and different regimes are studied and compared. As a benchmark, the efficient allocation is characterized (as e.g. implemented by an omniscient benevolent social planner) and then the focus is on the inefficiency of unfettered competitive markets calling for subsidies on cultural goods. Section 2 develops the model of consumption of cultural goods. Section 3 sets out the first-order conditions for welfare maximization with the consideration of dynamic aspects and characterizes an efficient allocation. Section 4 presents the underprovision of cultural capital in the laissez-faire economy with two alternative assumptions about consumer behavior. Section 5 compares the results from sections 3 and 4. Section 6 discusses the cultural policy options with Pigouvian tax/subsidy schemes. Section 7 concludes.

## 2. The Model

Suppose that in a simple economy two goods are produced and consumed over time:  $X$  is a cultural good and  $Y$  is a normal (composite) consumer good like e.g. food or clothing. Examples of cultural goods would be attending concerts, dramas, operas or visiting museums. The consumption of cultural good  $X$  over time leads to the accumulation of cultural capital,  $Z$ . There are  $n$  ( $n \geq 2$ ) identical consumers whose utility depends, at any point in time,  $t$ , on  $X_t, Y_t$  and  $Z_t$ :

$$u_t = U(X_t, Y_t, Z_t), \quad (1)$$

where  $U_X > 0$ ,  $U_Y > 0$ ,  $U_Z > 0$  and  $U_{XX} < 0$ ,  $U_{YY} < 0$ ,  $U_{ZZ} < 0$ ,  $U_{XY} \geq 0$ <sup>4</sup>. To simplify the exposition, the utility function (1) is assumed to be additively separable:

$$U(X_t, Y_t, Z_t) = U^1(X_t, Y_t) + U^2(Z_t). \quad (1')$$

We also assume that the function  $U$  is well-behaved in the sense that

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<sup>4</sup>  $U_X = \partial U / \partial X$ ,  $U_{XX} = \partial^2 U / \partial X^2$  etc. The subscript  $t$  for  $X, Y, Z$  denotes time.

$$\lim_{D \rightarrow 0} U_D^1 = \infty \text{ for } D = X, Y \text{ and } \lim_{Z \rightarrow 0} U_Z^2 = \infty. \quad (2)$$

As an implication, consumption bundles  $(X_t, Y_t)$  with  $X_t = 0$  or  $Y_t = 0$  for any  $t \geq 0$  will be ruled out along the optimal path (see below). The change in the stock of cultural capital over time is assumed to be:

$$\dot{Z}_t = \sum_{h=1}^n X_{ht} - \alpha Z_t, \text{ where } \dot{Z}_t := dZ_t / dt, \alpha > 0, \quad Z_t = Z_0 \geq 0 \text{ for } t = 0. \quad (3)$$

In (3),  $\dot{Z}_t$  is the net increase in the stock of cultural capital which is equal to the difference between the aggregate consumption of good  $X$  and depreciation  $\alpha Z_t$ . The constant rate of depreciation,  $\alpha$ , measures the instantaneous loss of cultural capital. Since all consumers are assumed to be identical, (3) is simplified by writing  $\sum_{h=1}^n X_{ht} = n X_t$ .

The model is completed by the simple linear production possibility constraint

$$nR = C_X n X_t + C_Y n Y_t \text{ (all } t \geq 0), \quad (4)$$

where  $nR$  is the time-invariant aggregate resource endowment and  $C_X, C_Y$  are positive time-invariant input-output coefficients. Using this framework we now characterize the efficient intertemporal allocation.

### 3. Allocative Efficiency

The social planner maximizes the present value of the consumers' aggregate utilities over an infinite time horizon and with a constant rate of time preference,  $\delta$ :

$$\max \int_0^{\infty} nU(X_t, Y_t, Z_t) e^{-\delta t} dt, \quad (5)$$

subject to (3) and (4). The optimal control problem (3) - (5) is solved by applying the current-value Hamiltonian

$$H = nU(X_t, Y_t, Z_t) + V(nX_t - \alpha Z_t) + W(nR - nC_X X_t - nC_Y Y_t). \quad (6)$$

The costate variable  $V$  is the shadow price of cultural capital measuring the value the social planner attaches to an increment of cultural capital. In other words,  $V$  measures the additional utility in the future which today's consumption  $X_t$  creates by raising the cultural capital. Hence it measures the value of the positive externality for future periods of today's consumption  $X_t$ .  $W$  is the Lagrange multiplier attached to equation (4). The first-order conditions for a solution to (5) are:

$$\frac{\partial H}{\partial X} = nU_X + nV - nWC_X = 0, \quad (7)$$

$$\frac{\partial H}{\partial Y} = nU_Y - nWC_Y = 0, \quad (8)$$

$$\dot{V} = \delta V - \frac{\partial H}{\partial Z} = (\alpha + \delta)V - nU_Z. \quad (9)$$

By combining the equations (7) and (8) we get

$$\frac{U_X + V}{U_Y} = \frac{C_X}{C_Y}, \quad \text{or, equivalently,} \quad \frac{U_X}{U_Y} = \frac{C_X}{C_Y} - \frac{V}{U_Y}, \quad (10)$$

and therefore  $\frac{U_X}{U_Y} - \frac{C_X}{C_Y} \Leftrightarrow V = 0$ . According to equation (10), each consumer's marginal

rate of substitution between the cultural good  $X$  and the consumption good  $Y$  must equal the marginal rate of transformation ( $\frac{C_X}{C_Y}$ ) minus the ratio of the shadow price of cultural capital

and the marginal utility with respect to consumption good  $Y$ , ( $\frac{V}{U_Y}$ ). Hence the rate at which

the agent is willing to give up of the consumption good  $Y$  to acquire the cultural good  $X$  depends on the shadow price of cultural capital  $V$ . The agent is willing to give up the more of consumption good  $Y$  to acquire an additional unit of cultural good  $X$  the greater is  $V$ .

Before we further characterize the optimal time path, it is convenient first to investigate the properties and the optimality of the steady states.

### 3.1. Steady States

In view of (1') we have  $U_Z(X, Y, Z) = U_Z^2(Z)$ . Hence equation (9) reads

$$\dot{V} = (\alpha + \delta) V - nU_Z^2(Z). \quad (11)$$

For  $\dot{V} = 0$ , equation (11) yields

$$V = \frac{nU_Z^2(Z)}{\alpha + \delta} =: nK(Z). \quad (12)$$

Since  $U_{ZZ} < 0$  by assumption, the  $\dot{V} = 0$  locus implied by equation (12) is negatively sloped.

Using equation (10),  $V$  can be rewritten as

$$V = \frac{C_X}{C_Y} U_Y^l(X, Y) - U_X^l(X, Y). \quad (13)$$

Substituting  $Y = \frac{R}{C_Y} - \frac{C_X}{C_Y} X$  from (4) into equation (13) we get

$$V = \frac{C_X}{C_Y} U_Y^l \left[ X, \left( \frac{R}{C_Y} - \frac{C_X}{C_Y} X \right) \right] - U_X^l \left[ X, \left( \frac{R}{C_Y} - \frac{C_X}{C_Y} X \right) \right] =: F(X). \quad (14)$$

The derivative of function  $F$  is:

$$F_X = 2 \frac{C_X}{C_Y} U_{XY} - \left( \frac{C_X}{C_Y} \right)^2 U_{YY} - U_{XX}. \quad (15)$$

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Owing to the assumptions on  $U$  introduced in equation (1),  $F_X$  is positive.

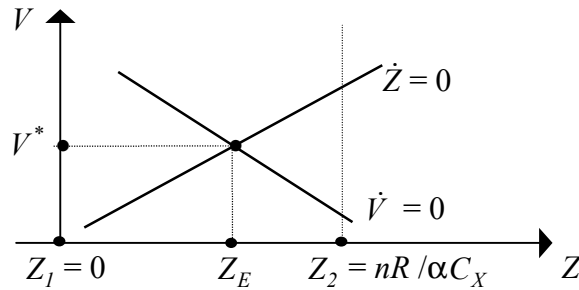
For  $\dot{Z} = 0$ , equation (3) yields  $X = \frac{\alpha Z}{n}$ . We insert this equation in function  $F$  from (14) to obtain the  $\dot{Z} = 0$  locus in the  $(V, Z)$  space.

$$V = F\left(\frac{\alpha Z}{n}\right) =: G(Z, n). \quad (16)$$

Hence along the  $\dot{Z} = 0$  locus  $V$  is strictly increasing in  $Z$ .

A steady state of the economy is defined by a situation  $\dot{Z}_t = \dot{X}_t = \dot{Y}_t = 0$  that prevails for all  $t$  following some point in time,  $t_0 \geq 0$ . Obviously, such a state is given at the value of cultural capital, denoted  $Z_E$  in Figure 1, that solves the equations (12) and (16) simultaneously:  $nK(Z) = G(Z, n)$ . Note that owing to (2) the solution  $Z_E$  to this equation is unique and satisfies  $Z_E > 0$ , because  $G_Z > 0$  and  $G(Z, n) > 0$  for large values of  $Z$  and because  $K_Z < 0$  and  $K(Z) > 0$  for all  $Z > 0$ . As a consequence,  $V_E = G(Z_E, n) > 0$  and  $X_E := \alpha Z_E / n > 0$ . We also conclude that  $Y_E > 0$  since owing to (2)  $F(X)$  is not defined for  $X = X_{max} = R / C_X$ . In the following we refer to  $(V_E, Z_E, X_E, Y_E)$  as the (unique) *interior* steady state.

Figure 1: The steady states of the economy

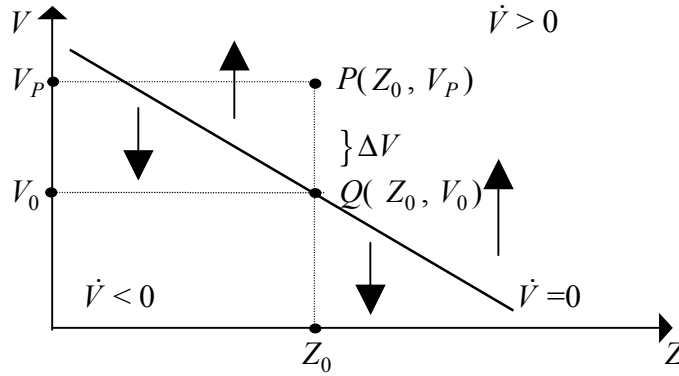


To see that the differential equations (3) and (9) also involve further steady states at the boundary, consider the state  $(V_2, Z_2, X_2, Y_2)$  satisfying  $\dot{V}_2 > 0$ ,  $V_2 > 0$ ,  $X_2 := \alpha Z_2 / n = R / C_X$  and hence  $Y_2 = 0$ . Increasing  $V$  creates a pressure via (10) to increase  $X_2$  which would violate (4), however (so that (10) does not hold as an equality anymore). Hence  $\dot{X} = 0$  and  $\dot{Z} = 0$ . Using similar arguments we identify another boundary state, say  $(V_1, Z_1, X_1, Y_1)$ , satisfying  $\dot{V}_1 < 0$ ,  $V_1 < 0$ ,  $X_1 = Z_1 = 0$  and  $Y_1 = R / C_Y$ . For convenience of later reference we call these states boundary steady states  $S1$  and  $S2$ , respectively.

### 3.2. Characterization of the optimal time path

To investigate the transitional dynamics of cultural capital and its shadow price, we now develop a phase diagram. The  $\dot{V} = 0$  line in Figure 2 separates two regions. To determine the direction of motion of  $V$  over time above and below that line, respectively, consider an arbitrary point on the  $\dot{V} = 0$  line, for example the point  $Q$  in Figure 2 whose coordinates are  $(Z_0, V_0)$ .

Figure 2: Direction of motion of  $V$



A deviation by  $\Delta V \neq 0$  from point  $Q$  gives

$$\dot{V} = (\alpha + \delta)(V_0 + \Delta V) - nU_Z^2(Z_0), \quad (17)$$

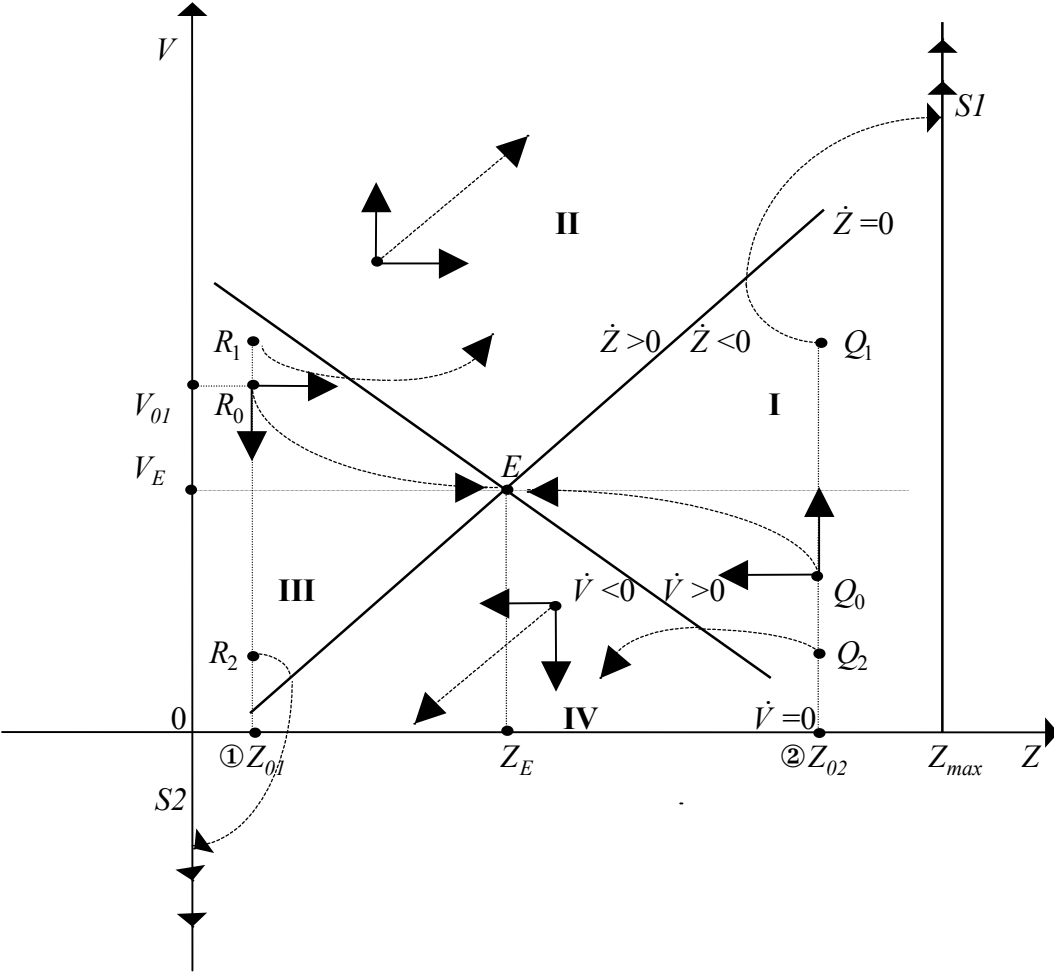
and it clearly follows that

$$\dot{V} = (\alpha + \delta)(V_0 + \Delta V) - nU_Z^2(Z_0) \begin{cases} > \\ = \\ < \end{cases} 0, \quad \text{if and only if} \quad \Delta V \begin{cases} > \\ = \\ < \end{cases} 0. \quad (18)$$

Consequently, for all points  $(Z, V)$  above the  $\dot{V} = 0$  locus,  $\dot{V}$  is positive, as indicated by the arrows pointing north in Figure 2. The opposite holds for all points  $(Z, V)$  below the  $\dot{V} = 0$  line. Consider next the locus of points for which  $\dot{Z} = 0$  (Figure 1). Using the same method, the dynamics of  $Z$  show that, for all points  $(Z, V)$  to the right of the locus,  $\dot{Z}$  is negative, and the arrows point west. Correspondingly, left to the  $\dot{Z} = 0$  locus the arrows point east and  $\dot{Z}$  is

positive. We now combine these dynamics of  $Z$  with the dynamics of  $V$  from Figure 2 to obtain Figure 3. The two isoclines partition the space into four regions, denoted by I, II, III and IV. Point  $E$  illustrates the unique interior steady state.

Figure 3: Phase diagram



In region I the direction of motion is northwest. There exists a path starting e.g. from  $Q_0$  in Figure 3 in this region, which leads right into  $E$ . If a starting point is chosen above or below the point  $Q_0$ , e.g. at point  $Q_1$  or  $Q_2$ , the system will never reach the steady state  $E$ . The trajectory starting at  $Q_1$  is not optimal, since it implies a growing accumulation of cultural capital until, eventually, the boundary steady state  $S2$  (as defined above) is reached. Since this state implies  $Y = 0$ , it is suboptimal owing to assumption (2). The trajectory starting at  $Q_2$  is not optimal either, since it implies that the cultural capital is eventually driven down to zero. This trajectory tends towards the boundary steady state  $S1$  (as defined above) which is also

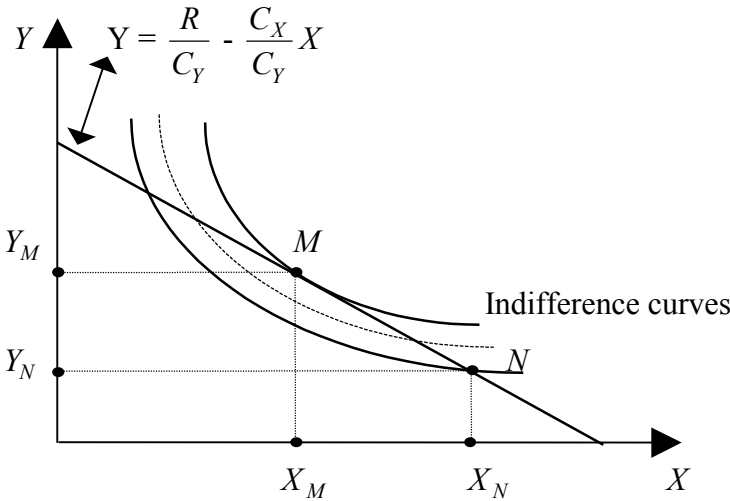


suboptimal in view of (2). In region II the arrows point northeast. All trajectories starting in this region will fail to reach the point  $E$ , and hence are suboptimal. Region III is analogous to region I. Here the arrows point southeast and therefore there exists an optimal path starting e.g. from  $R_0$  which leads to the steady state  $E$ . If the starting position is above or below  $R_0$ , the economy will be on a suboptimal path. In region IV the arrows point southwest, so that all trajectories starting there lead to the suboptimal boundary steady state  $S2$ . Regions I and III are the typical saddle-path cases. The trajectories passing through these regions either reach the optimal steady state  $E$  or suboptimal boundary steady states depending on where the starting positions are located.

So far our discussion showed that the optimal trajectory leads to the steady state  $E$ . Since  $V_E = G(Z_E, n) > 0$ , it follows from (9) that along the optimal path  $V_E > 0$  needs to hold for all  $t$ . An important implication of that observation is contained in (10). Since  $V > 0$  yields  $\frac{U_X}{U_Y} > \frac{C_X}{C_Y}$ , the optimal consumption bundle  $(X_t, Y_t)$  is represented by a point such as  $N$  in

Figure 4. In that point, the agent consumes more of the cultural good and less of good  $Y$  than she would have chosen in a world where the consumption of cultural goods does not affect the generation of cultural capital (or, alternatively, in a world where all agents ignore the built-up of cultural capital through the consumption of cultural goods).

Figure 4: The shadow price  $V$  and consumption possibilities



We now investigate in more detail the time pattern of optimal cultural goods consumption, cultural capital and its shadow price focusing in particular, on how that time pattern depends

on the economy's initial endowment of cultural capital. For that purpose observe that the total derivative of function  $V$  from equation (14) with respect to time yields  $\dot{X} = \frac{I}{F_X} \dot{V}$ . Hence  $\text{sign } \dot{X} = \text{sign } \dot{V}$ . Moreover, the total derivative of equation (5) with respect to time yields  $C_Y \dot{Y} = -C_X \dot{X}$ . Hence  $\text{sign } \dot{Y} = (-1) \text{sign } \dot{X}$ . We also make use of the information from equation (3) that  $\dot{Z} = X - \alpha Z$ , and from equation (11) that  $\dot{V} = (\alpha + \delta) V - nU_Z^2(Z)$ .

**Situation ①: The initial endowment of cultural capital is relatively small**

Suppose the initial stock  $Z_{01}$  of cultural capital in Figure 3 is smaller than the steady state stock  $Z_E$ . Then the optimal trajectory towards the steady state  $E$  is characterized by

$$\dot{X}_t < 0, \dot{Y}_t > 0, \dot{Z}_t > 0 \text{ and } \dot{V}_t < 0 \text{ for all } t \geq 0.$$

Through the consumption of cultural goods, cultural capital will be accumulated. With the relatively low initial stock of cultural capital,  $Z_{01}$ , the socially optimal policy is to set the initial shadow price of cultural capital at  $V_{01} > 0$  well above its steady state level  $V_E$ , because the trajectory starting from point  $R_0$  with coordinates  $(Z_{01}, V_{01})$  leads to the steady state  $E$ . Before the steady state  $E$  is reached, the optimal  $X_t$  is greater than the steady state value  $X_E = F^{-1}(V_E)$  and  $X_E$ , in turn, is greater than the consumption  $X_M$  in Figure 4 where  $X_M$  would be utility maximizing in an economy in which the consumption of the cultural good that does not affect the generation of cultural capital.

**Situation ②: The initial endowment of cultural capital is relatively large**

Suppose the initial value  $Z_{02}$  of cultural capital is greater than the steady state value  $Z_E$ . Then for all  $t \geq 0$  the optimal trajectory towards the steady state is characterized by

$$\dot{X}_t > 0, \dot{Y}_t < 0, \dot{Z}_t < 0 \text{ and } \dot{V}_t > 0 \text{ for all } t \geq 0.$$

Since we showed above that  $V_t > 0$  for all  $t$  along the optimal path, it is obviously true that  $V_t > 0$  for  $t = 0$ . This observation is quite remarkable since it proves wrong the plausible intuition that in case of a *very* large initial capital stock  $Z_{02}$ , it might be optimal to reduce

consumption of cultural goods below  $X_M$  in an initial phase (implying  $V < 0$ ) to speed up the reduction of cultural capital towards its steady state level.

## 4. Underprovision of Cultural Capital in the Laissez-faire Economy

We now study the intertemporal allocation of cultural goods and cultural capital in the laissez-faire competitive economy with special emphasis on how consumers behave in this regime. The results will turn out to depend on our assumption about the consumers' behavior. First we assume that all consumers maximize utility taking as given the prevailing stock of cultural capital (and hence ignoring the effect of the consumption of cultural goods on cultural capital altogether). This assumption appears to be realistic in case of very large numbers of consumers,  $n$ . As an alternative we will assume Cournot-Nash behavior that is particularly plausible if  $n$  is not too large. A Cournot-Nash consumer maximizes her utility taking as given the other agents' consumption of the cultural good and accounting for the impact of her own cultural good consumption on the change in the stock of cultural capital. In what follows both models are successively investigated.

### 4.1. Cournot-Nash consumers

Consumer  $h$  solves the optimization problem<sup>5</sup>:

$$\max_{\{X_t, Y_t\}} \int_0^{\infty} U(X_t, Y_t, Z_t) e^{-\delta t} dt, \quad (19)$$

subject to

$$\dot{Z}_t = X_t + \bar{X} - \alpha Z_t, \text{ and} \quad (20)$$

$$R \geq P_X X_t + P_Y Y_t, \quad (21)$$

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<sup>5</sup> Subscript  $h$  is suppressed to avoid clutter.

In (20)  $\bar{X} := \sum_{j \neq h} X_{jt}$  is considered constant by consumer  $h$ , and in (21) we set  $P_X = C_X$ ,

$P_Y = C_Y$  and  $P_R \equiv 1$ . The optimal control problem (19) - (21) is solved by applying the current-value Hamiltonian:

$$H = U(X_t, Y_t, Z_t) + \mu (X_t + \bar{X} - \alpha Z_t) + \lambda (R - P_X X_t - P_Y Y_t). \quad (22)$$

The variable  $\mu$  is the shadow price of cultural capital.  $\lambda$  is the Lagrange multiplier attached to (21). The first-order conditions for a solution to (19) are:

$$\frac{\partial H}{\partial X} = U_X + \mu - \lambda P_X = 0, \quad (23)$$

$$\frac{\partial H}{\partial Y} = U_Y - \lambda P_Y = 0, \quad (24)$$

$$\dot{\mu} = \delta \mu - \frac{\partial H}{\partial Z} = (\alpha + \delta) \mu - U_Z. \quad (25)$$

Combining equations (23) and (24) we obtain

$$\frac{U_X}{U_Y} = \frac{P_X}{P_Y} - \frac{\mu}{U_Y}. \quad (26)$$

Similar as (10), equation (26) tells us that each consumer's marginal rate of substitution between good  $X$  and good  $Y$  must equal the price ratio minus the ratio of the shadow price of cultural capital and the marginal utility with respect to good  $Y$ . Combining equations (25) and

(26) and substituting  $Y = \frac{R}{P_Y} - \frac{P_X}{P_Y} X$  from (21)  $\mu$  can be rewritten as

$$\mu = \frac{P_X}{P_Y} U_Y \left[ X, \left( \frac{R}{P_Y} - \frac{P_X}{P_Y} X \right) \right] - U_X \left[ X, \left( \frac{R}{P_Y} - \frac{P_X}{P_Y} X \right) \right] =: F(X). \quad (27)$$

Since all consumers are assumed to be identical, it is appropriate to restrict the equilibrium analysis to a symmetric Cournot-Nash equilibrium. Hence for every agent there exists an

individually optimal time path  $\{X_t\}$ ,  $\{Y_t\}$ ,  $\{Z_t\}$  for any given  $\bar{X}$ . The associated necessary equilibrium condition is then  $X_i = X_j = X$  for all  $i, j = 1, \dots, n$ . It follows that

$$\sum_{j \neq h} X_j = (n - 1) X. \quad (28)$$

As before, the change in the stock of cultural capital is given by

$$\dot{Z} = n X - \alpha Z. \quad (29)$$

The equations (20) and (29) are not contradictory since (20) is the individual consumer's perception whereas (29) reflects the actual intertemporal changes of cultural capital under the (necessary) equilibrium condition  $X_j = X_h = X$  for all  $j, h = 1, \dots, n$ . We combine (29) and (27) to obtain the  $\dot{Z} = 0$  locus in the  $(\mu, Z)$  space:

$$\mu = F(X) = F\left(\frac{\alpha Z}{n}\right) =: G(Z, n). \quad (30)$$

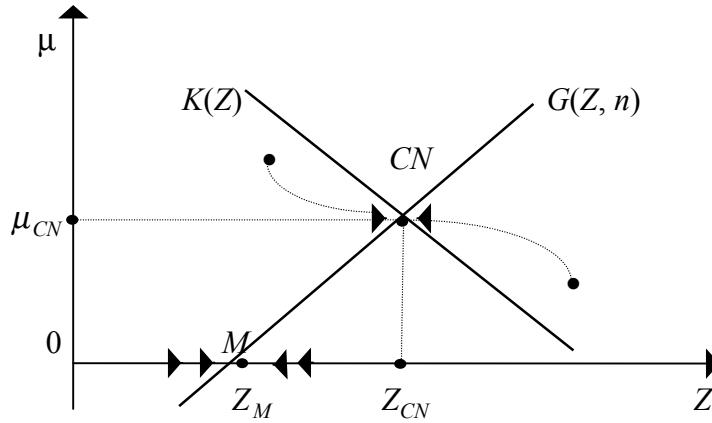
For  $\dot{\mu} = 0$  equation (25) yields

$$\mu = \frac{U_Z(Z)}{\alpha + \delta} =: K(Z). \quad (31)$$

It follows that the time path of the Cournot-Nash equilibrium tends towards an (interior) steady state characterized by the stock of cultural capital,  $Z_{CN} > 0$ , implicitly defined through the equation  $G(Z, n) = K(Z)$  and illustrated in Figure 5. We also know that  $\mu_{CN} := G(Z_{CN}) > 0$  and hence the equilibrium dynamics are qualitatively the same as in case of the optimal time path discussed in section 3.2.

If the initial endowment of cultural capital happens to be  $Z_0 > Z_{CN}$ ,  $Z$  will successively shrink until  $Z_{CN}$  is eventually reached. Conversely, if  $Z_0 < Z_{CN}$ , cultural capital accumulates until the steady state  $Z_{CN}$  is attained. The comparison between the Cournot-Nash time path and the optimal path is postponed to section 5.

Figure 5: Cultural capital in the laissez-faire economy



#### 4.2. Ignorant consumers

Suppose now the number of consumers  $n$  is very large. Then it is plausible to assume that each consumer ignores the positive impact her cultural good consumption (through its contribution to the accumulation of cultural capital) has on both her own and all other agents' utilities. In formal terms, now the representative consumer maximizes (19) with regard to  $X_t$  and  $Y_t$  subject to (20) taking the “prevailing“ level of cultural capital as given. The “law of motion“ of the stock of cultural capital is still determined by (3) but disregarded by all consumers. In other words, the time path of cultural capital  $\{Z_t\}$  depends on the time path  $\{X_t\}$  of cultural good consumption, as before, but that relation does not play a role in the individual optimization problems. Translated into our formal analysis this behavioral assumption is expressed by setting the shadow price  $\mu$  equal to zero. As a consequence, the Hamiltonian (22) degenerates to:

$$H = U(X_t, Y_t, Z_t) + \lambda(R - P_X X - P_Y Y). \quad (32)$$

The first-order conditions are:

$$\frac{\partial H}{\partial X} = U_X - \lambda P_X = 0, \quad (33)$$

$$\frac{\partial H}{\partial Y} = U_Y - \lambda P_Y = 0, \quad (34)$$

By combining the (33) and (34) we obtain

$$\frac{U_X}{U_Y} = \frac{P_X}{P_Y}. \quad (35)$$

According to equation (35) each consumer's marginal rate of substitution between the cultural good and good  $Y$  equals the price ratio at each point in time along the equilibrium path. This condition is satisfied in point  $M$  in Figure 4.

Since  $\mu_t \equiv 0$  for all  $t$  by assumption (implying  $\dot{\mu}_t = 0$  for all  $t$ ) the steady state is now fully characterized by  $\dot{Z} = 0$ . In terms of Figure 5 the  $\dot{Z} = 0$  locus shrinks into the point  $Z_M$ . Since each consumer chooses  $X_t = X_M$  for all  $t$  (with  $X_M$  as specified in Figure 4),  $Z_M := n X_M / \alpha$  is clearly the steady state value of cultural capital. Hence the phase diagram of Figure 5 degenerates to changes in the stock of cultural capital along the abscissa ( $Z$ -axis). If the initial endowment of cultural capital happens to be  $Z_0 > Z_M$ , cultural capital  $Z$  will gradually shrink until  $Z_M$  is eventually reached. Conversely, if  $Z_0 < Z_M$ , cultural capital accumulates until the steady state  $Z_M$  is attained.

## 5. Inefficiency of the Laissez-faire Market Economy

We now aim at comparing the cultural capital formation for the regimes:

- E.** Centralized economy with an omniscient benevolent social planner
- CN.** Market economy with Cournot-Nash consumers
- M.** Market economy with ignorant consumers

Table I: Steady states of different regimes

	Steady state values	Characterization of steady state
<b>E</b>	$V_E, Z_E, X_E$	$\dot{V} = 0, \dot{Z} = 0, \dot{X} = 0$ ; $V_E, Z_E$ and $X_E$ satisfy $V_E = n K(Z_E), V = G(Z_E, n)$ and $X_E = \alpha Z_E / n$
<b>CN</b>	$\mu_{CN}, Z_{CN}, X_{CN}$	$\dot{\mu} = 0, \dot{Z} = 0, \dot{X} = 0$ ; $\mu_{CN}, Z_{CN}$ and $X_{CN}$ satisfy $\mu_{CN} = K(Z_{CN}), \mu = G(Z_{CN}, n)$ and $X_{CN} = \alpha Z_{CN} / n$
<b>M</b>	$\mu_M \equiv 0, Z_M, X_M$	$\dot{\mu} = 0, \dot{Z} = 0, \dot{X} = 0$ ; $Z_M$ and $X_M$ satisfy $G(Z_M, n) = 0$ and $X_M = \alpha Z_M / n$

The characteristics of the pertinent steady states are summarized in Table I.

To see how the steady state values of the three regimes are related to each other, observe that the steady state capital stock associated to regimes  $j = E, CN, M$ , respectively, is determined by the equation

$$m_j K(Z) = G(Z, n), \quad (36)$$

if and only if  $m_E = n \geq 2$ ,  $m_{CN} = 1$  and  $m_M = 0$ . Since  $K(Z) > 0$  for all  $Z > 0$ ,  $K_Z < 0$  and  $G_Z > 0$ , it is straightforward that  $Z_M < Z_{CN} < Z_E$ . Using the steady state condition  $X = nZ / \alpha$  these inequalities yield immediately  $X_M < X_{CN} < X_E$ , and (4) gives us  $Y_M > Y_{CN} > Y_E$ .

To illustrate these results, we combine the Figures in 3, 4 and 5 to obtain Figure 6.

If the social planner maximizes the consumers' aggregate utilities taking into account that the cultural goods consumption affects the generation of cultural capital, the optimal steady state is determined by the intersection of the graphs of  $G(Z, n)$  and  $nK(Z)$ . The associated cultural capital value is  $Z_E$  as shown in quadrant **I** of Figure 6. By drawing the graph of the steady state equation  $Z = \frac{\alpha X}{n}$  into quadrant **IV** of Figure 6 we determine the cultural goods consumption  $X_E$  corresponding to the cultural capital  $Z_E$ . The consumer's steady state consumption  $(Y_E, X_E)$  is illustrated by point  $N$  in quadrant **III**.

If the consumers exhibit Cournot-Nash behavior, the steady state value of cultural capital  $Z_{CN}$  is determined by solving  $K(Z) = G(Z, n)$  as shown in quadrant **I**. The consumers choose the consumption bundle  $(Y_{CN}, X_{CN})$  corresponding to point  $P$  in quadrant **III**. In a market economy with ignorant consumers, a steady state is reached at point  $Z_M$  as shown in quadrant **I** of Figure 6. In this case, the consumers choose the consumption bundle  $(Y_M, X_M)$  as illustrated by point  $M$  in quadrant **III**.

To derive the relation between the shadow price of cultural capital,  $V$ , and consumption of good  $Y$  as shown by the negatively sloped curve of quadrant **II** recall from (14) and (16) that

$$V = G(Z, n) = F\left(\frac{\alpha Z}{n}\right) = F(X). \text{ Moreover, (4) implies } X = \left(\frac{R}{C_X}\right) - \left(\frac{C_Y}{C_X}\right)Y =: H(Y). \text{ Hence}$$

$V = D(Y) := F[H(Y)]$  with  $D_Y := F_X H_Y < 0$ . The higher the shadow price of cultural capital is, the less of consumer good  $Y$  will be consumed.

Figure 6: Comparison of steady states of different regimes



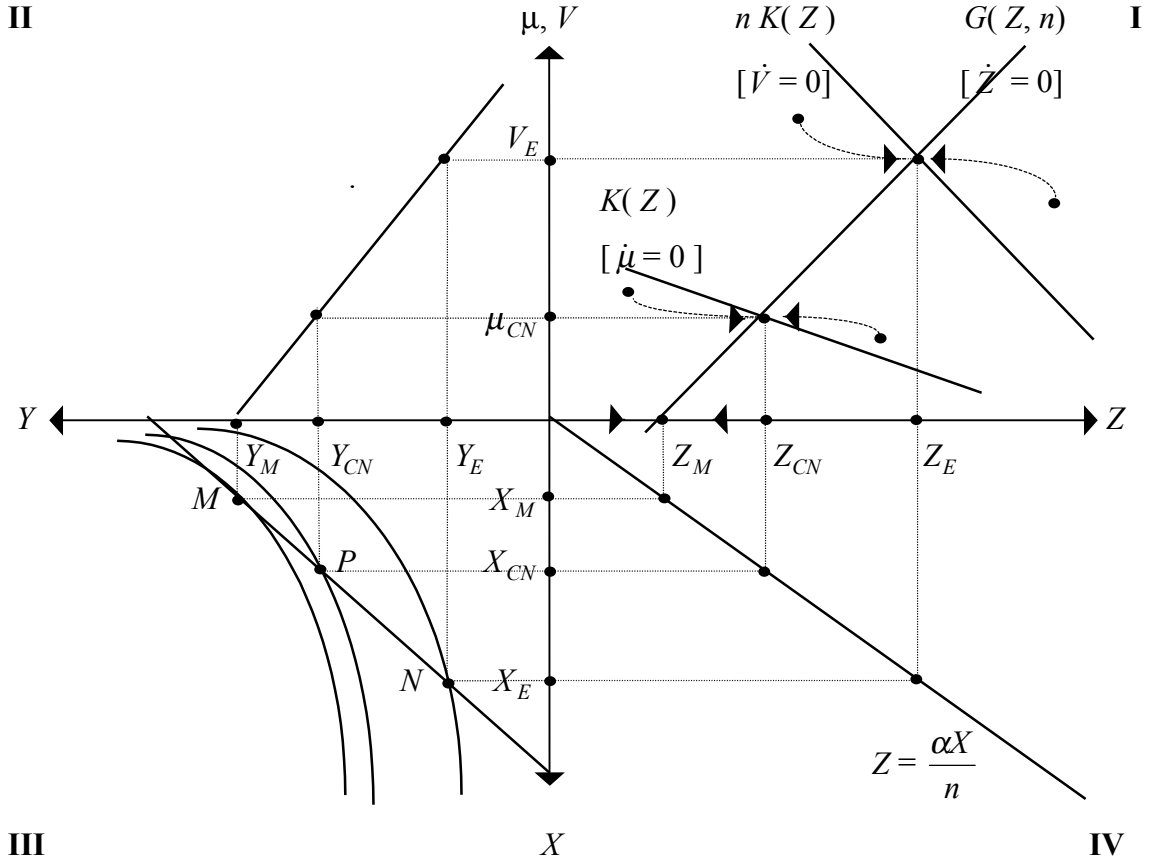


Table II: The impact of the numbers of consumers on the steady state value of cultural capital

	<i>E</i>	<i>CN</i>	<i>M</i>
$\frac{dZ}{dn}$	$-\frac{G_n - K(Z)}{G_Z - nK_Z}$	$-\frac{G_n}{G_Z - K_Z}$	$-\frac{G_n}{G_Z}$

After having characterized and compared the steady states associated to the regimes *E*, *CN* and *M* we conclude the present section by specifying the comparative dynamics regarding the impact of exogenous changes in the number of consumers,  $n$ , on the respective steady states. Total differentiation of the steady state condition (36) gives us the results listed in Table II. We find that the steady state value of cultural capital increases in all regimes as the number of consumers gets larger. Unfortunately, comparisons across regimes are not feasible, in general, since the partial derivatives composing the terms listed in Table II are evaluated at different values of  $Z$  ( $Z_E > Z_{CN} > Z_M$ ). Suppose, however, the functions  $G$  and  $K$  happen to have constant partial derivatives (approximately, at least). The Table II implies

$$\left(\frac{dZ}{dn}\right)_E > \left(\frac{dZ}{dn}\right)_{CN} > \left(\frac{dZ}{dn}\right)_M$$

In that case, the underprovision of cultural capital in the market economy would be the more pronounced the larger is the society under consideration, and the market economy with ignorant consumers fares worse than the economy with Cournot-Nash consumers.

## 6. Pigouvian Subsidies on the Consumption of Cultural Goods

We showed in the previous section that in the absence of cultural policy the market economy provides cultural goods and cultural capital at inefficiently low levels. It is therefore natural to think about subsidizing cultural goods as a means to stimulate demand for eliminating the distortion. In formal terms a subsidy turns the consumer's budget constraint (21) into

$$R \geq T + (P_X - S_X) X_t + P_Y Y_t. \quad (37)$$

Where  $S_X$  is the subsidy rate and  $T$  is a lumpsum tax taken as given by the consumers and set by the government such as to finance the total subsidy on cultural goods. In section 5 the extent of market failure was shown to depend on consumers behavior. Hence the rate of the corrective subsidy must take the consumers' actual behavior into account. It is therefore necessary to investigate the appropriate subsidies for each type of market economies starting with Cournot-Nash behavior of consumers.

### 6.1 Cournot-Nash consumers

Taking (37) into account, the Hamiltonian (22) is modified to read

$$H = U(X_t, Y_t, Z_t) + \mu (X_t + \bar{X} - \alpha Z_t) + \lambda [R - T - (P_X - S_X) X_t - P_Y Y_t]. \quad (38)$$

The first-order conditions for a solution to (38) yield:

$$\frac{U_X}{U_Y} = \frac{P_X - S_X}{P_Y} - \frac{\mu}{U_Y}. \quad (39)$$

Denote by  $S_X^{CN}$  that particular value of the subsidy that internalizes the cultural externality. It is implicitly defined by setting equal (10) and (39):

$$\frac{U_X}{U_Y} = \frac{P_X - S_X^{CN}}{P_Y} - \frac{\mu}{U_Y} = \frac{P_X}{P_Y} - \frac{V}{U_Y}. \quad (40)$$

Solving (40) for  $S_X^{CN}$  yields:

$$S_X^{CN} = \frac{P_Y(V - \mu)}{U_Y}. \quad (41)$$

In view of (41)  $S_X^{CN}$  is essentially the difference between the social value of cultural capital,  $V$ , and its private value,  $\mu$ , where  $P_Y / U_Y$  is used to turn the dimension ‘‘utility’’ into the dimension ‘‘money’’. Using (12) and (31), equation (41) can be transformed into

$$S_X^{CN} = \frac{P_Y U_Z}{U_Y} \frac{(n-1)}{\alpha + \delta}. \quad (42)$$

According to (42) the subsidy rate  $S_X^{CN}$  depends on the number of consumers,  $n$ , the depreciation rate of cultural capital,  $\alpha$ , the time preference,  $\delta$ , and on the marginal willingness-to-pay for cultural capital,  $(U_Z/U_Y)$ . The greater is  $n$ , the higher is the efficient subsidy rate, *ceteris paribus*. The faster the cultural capital depreciates, the lower is the efficient subsidy rate. The higher the agents’ preference for present consumption, the lower is the efficient subsidy rate.

(42) has also another interesting interpretation: Suppose through her consumption of the cultural good, consumer  $h$  increases the cultural capital at the margin. Since  $h$  is a Cournot-Nash consumer, she takes the beneficial effect of this change on her own well-being into account and evaluates that effect at  $\mu = U_Z / (\alpha + \delta)$ . Owing to symmetry the same beneficial effect accrues to all other consumers. But since no market transaction is involved,  $h$  does not receive any remuneration for her contribution to the other consumers’ well-being and therefore ignores these (external) benefits in her own utility maximization calculus. The

subsidy rate  $S_X^{CN}$  from (42) is designed to pay consumer  $h$  exactly the value of her (previously external) benefits to the  $(n - 1)$  other consumers. Thus the cultural externality is internalized.

## 6.2 Ignorant consumers

Using (38), the Hamiltonian from equation (22) is now:

$$H = U(X_t, Y_t, Z_t) + \lambda [R - (P_X - S_X)X_t - P_Y Y_t - T]. \quad (43)$$

The first-order conditions for a solution to (43) yield:

$$\frac{U_X}{U_Y} = \frac{P_X - S_X}{P_Y}. \quad (44)$$

Denote by  $S_X^M$  the efficient subsidy rate which is characterized, in view of (10), (12), (31), (42) and (44), as

$$S_X^M = \frac{P_Y V}{U_Y} = \frac{P_Y U_Z}{U_Y} \frac{n}{\alpha + \delta} = S_X^{CN} + \frac{P_Y U_Z}{U_Y (\alpha + \delta)}. \quad (45)$$

Comparing (45) and (42),  $S_X^M$  and  $S_X^{CN}$  turn out to be quite similar. Since the terms  $\frac{P_Y U_Z}{U_Y}$

and  $\frac{1}{\alpha + \delta}$  are the same in both cases, it follows that  $S_X^M > S_X^{CN}$ .

This observation is straightforward to interpret: The ignorant consumer ignores the beneficial effect of an increase in cultural capital, induced by herself, on all other consumers. In this respect she behaves like the Cournot-Nash consumer, and to internalize that externality she needs to be subsidized by the rate  $S_X^{CN}$ . But in contrast to the Cournot-Nash consumer the ignorant consumer also ignores in her consumption plan the benefits she offers to herself through an increase in cultural capital induced by her own cultural good consumption. The value of this benefit is  $\mu = U_Z / (\alpha + \delta)$ , in terms of utility, or,  $P_Y U_Z / [U_Y (\alpha + \delta)]$ , in terms of money. The latter (money) value must be added to the subsidy rate  $S_X^{CN}$  to achieve a

complete internalization of the cultural externality in a market economy with ignorant consumers.

A final remark is in order regarding the comparison of  $S_X^M$  and  $S_X^{CN}$  when the number of consumers becomes very large. With increasing  $n$ , the difference  $S_X^M - S_X^{CN}$  remains positive but tends to zero. The greater is  $n$  the smaller is the impact of each individual Cournot-Nash consumer on the formation of cultural capital and the less significant becomes the difference between Cournot-Nash behavior and ignorant behavior. In that sense ignorant behavior may be considered a fairly good approximation for Cournot-Nash behavior in sufficiently large societies.

## 7. Concluding Remarks

The justifications for government support of cultural goods discussed in the cultural economics literature are mostly confined to static analysis. The present paper demonstrated that the consideration of dynamic aspects of positive consumption externalities makes a strong, and in our view, more convincing case for government subsidization. The preceding analysis builds on a simple game theoretic model and finds that, in the absence of any government intervention, both cultural goods and cultural capital are underprovided and that this allocative inefficiency can be eliminated by an appropriate subsidy on the consumption of cultural goods.

Essentially, this conclusion is driven by the basic hypothesis that the consumption of cultural goods is not only beneficial for the individual consumer but contributes to form a “better“ or a “more civilized“ society which is enjoyed by all its members irrespective of (and in addition to) their own cultural good consumption. Therefore, the empirical relevance of that approach depends heavily on the concept of cultural capital and its measurability. Similar as with the related notion “social capital“ or even “human capital“, empirical measurement turns out to be difficult which, unfortunately, leaves us without clear-cut evidence for the hypothesis that citizens appreciate the accumulation of cultural capital.

While this ambiguity is clearly unsatisfactory, it is not convincing to argue, on the other hand, that a concept is rendered elusive and (hence) useless whenever it is difficult to measure. Consider, as a case in point, the notion of merit goods that was introduced some fifty years ago and is heavily disputed since then. The present model may be interpreted as a

rationalization of the issues involved in the allocation of a good which some people use to call a merit good. One could argue that our modelling of the cultural good as a merit good is as unsatisfactory as the ad hoc merit good approach because the “black box“ merit good is merely substituted by a new “black box“ called cultural capital. We maintain, however, that our model focuses on a well-defined externality issue and it draws our attention on a specific hypothesis about cultural capital, its development and effects rather than on a paternalistic merit-good argument.

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