Dynamic Macroeconomics

Problem Set 4

1. **Computing growth rates** Consider the data given in the following table on the level of real GDP per capita for China, Germany and the United States.

Country	1980	2009
China	1183.72	7430.75
Germany	21415.04	32487.10
United States	25090.21	41101.86

Notes: Downloaded form the Penn World Table at http://pwt.econ.upenn.edu/. Entries are real GDP per capita. All values expressed in units of 2005 US Dollars.

Based on this data, answer the following questions.

a) How long does it take for the GDP of the three countries to double?

b) How long will it take (starting in 2009) for the level of GDP in China to catch up with the one in the United States?

2. **Golden rule saving rate** Consider the following version of the Solow model of a closed economy without population or technology growth.

Firms maximize their profits which are given by

$$\Pi_t = AK_t^{\alpha}N_t^{1-\alpha} - w_tN_t - R_tK_t.$$

by choosing N_t and K_t taking the factor prices w_t and R_t as given.

a) Find the first-order conditions characterizing optimal firm behavior.

b) Use the results from a) to show that $w_tN_t/Y_t = 1-\alpha$, where $Y_t = AK_t^{\alpha}N_t^{1-\alpha}$.

Households are endowed with labor and own the capital stock, which they lend to the firms on a period-by-period basis. Firms remit all of their profits to the households so that the budget constraint of households is given by

$$C_t + I_t = w_tN_t + R_tK_t + \Pi_t \tag{1}$$

c) Show that the right-hand side of (1) is equal to Y_t.

Capital accumulates over time according to $K_{t+1} = I_t + (1-\delta)K_t$, where I_t denotes investment. Assume that households consume a fraction $1-s$ of their income in each period and supply $N_t = N = 1$ units of labor in each period.

d) Write down a difference equation relating K_{t+1} to K_t and parameters of the model.

e) Plot K_{t+1} against K_t. Use the graph to argue that there is a steady-state with $K_{t+1} = K_t = \bar{K}$.

f) Solve for steady-state levels of capital, production and consumption. Discuss how they depend upon parameter values.

g) What saving rate s would maximize steady-state production? Discuss.

h) What saving rate s would maximize current period consumption? Discuss.

i) What saving rate s would maximize steady-state consumption? Discuss.

j) Suppose $\alpha = 0.33$. Currently the saving rate in the United States is somewhere between 10 and 15 percent. Would you recommend to increase the saving-rate to the one found in question i)?

3. **Simulation of the Solow Model** Consider the following equations

$$\begin{aligned} Y_t &= K_t^\alpha (A_t N_t)^{1-\alpha} \\ K_{t+1} &= I_t + (1-\delta)K_t \\ C_t + I_t &= Y_t \\ I_t &= sY_t \\ N_{t+1} &= (1+g_n)N_t \\ A_{t+1} &= (1+g_a)A_t \end{aligned}$$

and define $\hat{k}_t = K_t/A_tN_t$.

a) Derive the following equation

$$(1+g_a)(1+g_n)\hat{k}_{t+1} = s\hat{k}_t^\alpha + (1-\delta)\hat{k}_t.$$

b) Derive the steady-state value of $\hat{k}$ and discuss how it depends upon the parameters of the model.

c) Calculate the steady-state growth rates of Y and Y/N.

Form now on suppose that $g_n = 0.01$, $g_a = 0.02, \delta = 0.1, \alpha = 0.33, s = 0.2, A_0 = 1, N_0 = 1$.

d) Suppose that for periods 1-9 $\hat{k}$ is equal to its steady-state value. Then, in period $t = 10$, the saving rate increases to $s = 0.25$. Discuss what happens.

e) Calculate the growth rate of production in the periods after the change in the saving rate.

4. **Growth accounting** Assume that the aggregate production function takes the form $Y_t = A_t K_t^\alpha N_t^{1-\alpha}$ or expressed in logarithms

$$\ln Y_t = \ln A_t + \alpha \ln K_t + (1-\alpha)\ln N_t$$

Taking first difference yields

$$\Delta \ln Y_t = \Delta \ln A_t + \alpha\Delta \ln K_t + (1-\alpha)\Delta \ln N_t$$

where Δ indicates the first difference (i.e. $\Delta Y_t = Y_t - Y_{t-1}$). On the course web-site you can find a data set with (quarterly) information on real GDP (Y_t), hours worked in the non-farm sector (N_t) and real gross private investment (I_t). Assume throughout that $\alpha = 0.33$.

a) As a first step, construct a time series for the capital stock using the equation

$$K_{t+1} = I_t + (1-\delta)K_t$$

where $\delta = 0.03$. To set an initial value of K_0 use the formula

$$K_0 = \frac{1}{\delta}\frac{\bar{I}}{\bar{Y}}Y_0$$

where $\bar{I}$ is average investment in the first eight periods and $\bar{Y}$ is average GDP in the first eight periods of the data.

b) Compute time series on $\Delta \ln Y_t, \Delta \ln K_t$ and $\Delta \ln N_t$. Then compute the time series of $\Delta \ln A_t$.

c) Compute the accumulated logarithm of A_t by computing the cumulative sum of $\Delta \ln A_t$. Plot the resulting series. Discuss.

d) Calculate averages for each decade (i.e. $1950-1959, 1960-1969, ...$) of annual GDP growth rates and annual technology growth rates and display them in a table.