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PROBLEMS OF IRREVERSIBILITY IN THE CONTROL OF PERSISTENT POLLUTANTS

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ABSTRACT

This paper deals with two different types of irreversibility: Pollution irreversibility occurs if the stock of pollution cannot be reduced (any more) by nature's assimilative capacity; characteristics irreversibility occurs when some characteristic of the environmental resource is irreversibly lost as soon as the environmental degradation exceeds a certain threshold level. For both cases the optimal intertemporal strategy of pollution control is characterised under conditions of certainty. Then the issue of characteristics uncertainty is further investigated under the assumption that the social evaluation of the resource characteristic is uncertain. Moreover, if there is the prospect of better information at some future point in time, it is shown that an unwarranted anti-preservation bias in pollution management is introduced by the decision maker if he or she ignores the future emergence of new information. A numerical example clarifies the value of new information, known in the literature as the quasi-option value.

1. INTRODUCTION

The literature on irreversible destruction of natural or cultural resources followed two main routes, so far. The first is species extinction by (over)harvesting, which may be the result of optimising [Smith 1977, Sinn 1982] or not. The second is the issue of natural resource destruction by projects of industrial development with the Hells Canyon case as a prototypical example [Krutilla and Cicchetti 1972]. Both lines of reasoning are relevant for the problem at hand, but the framework of analysis they provide is not appropriate for studying irreversibilities caused by **gradually increasing accumulation** of pollutants in environmental resources.

To some extent, the environment is a renewable resource owing to its capacity of assimilating harmful pollutants. Scientists tell us, however, that nature's assimilative capacity is limited: It is very low or absent, for practical purposes, in the case of some pollutants, and when it is positive, it eventually tends to zero with increasing ambient pollution. Therefore, pollutants can be considered to be the more persistent, the smaller is the respective assimilative capacity. The present paper focuses on the polar case of **strongly persistent pollutants** in the sense that nature provides no assimilative services at all. In other words environmental pollution is irreversible by assumption. The zero-assimilation restriction will considerably simplify the subsequent analysis, but it will not, by any means, render trivial the decisions on intertemporal optimal pollution control.

In addition to that **pollution irreversibility** the paper addresses another type of irreversibility - denoted **characteristic irreversibility**, for short – that occurs when some characteristic of the environmental resource under review is irreversibly lost as soon as, for the very first time, the environmental degradation exceeds a certain threshold value. For example, pollution may cause irreversible modifications for the habitat of plants or animals leading to species extinction or it may cause the destruction of "cultural characteristics" like the Akropolis in Athens.

Section 2 introduces the model and characterises the optimal strategy of irreversible pollution under certainty. It is shown that the optimal pollution control typically implies a time path with decreasing emission and decreasing environmental quality tending towards an unique steady state. Section 3 continues to deal with optimal control under certainty, but focuses on the impact of irreversible losses of resource characteristics. Such a loss is assumed to occur, whenever the environmental quality drops below some critical threshold value. Technically, the loss introduces a shift in the evaluation function implying that now the optimal control also depends on the comparison of total conditions. It turns out that now the optimal path is very sensitive to the magnitude of the parametric shift caused by the destruction of the resource characteristic.

Section 4 extends the issue of destroying or preserving a resource characteristic to the case of uncertainty about the value of that characteristic. Taking expected welfare as the relevant objective, uncertainty does not change the results of section 3 conceptually. A marked difference emerges, however, when we introduce the additional assumption, following Weisbrod [1964] and others, that there is the **prospect of better information** about the social evaluation of the resource characteristic at some future point in time. Now the question arises as to whether the resource characteristic should be preserved, at least as long as the new information becomes available. Is sequential decision making of this kind a more appropriate planning procedure than a once-and-for-all decision? Is there a bias in favor of or against preservation when in today's decision the emergence of future information is properly taken into account?

The literature on that issue deals with the concepts of "option value" and "quasi-option value" in the context of industrial development. It is shown that the basic arguments put forward in this literature carry over to the problem under review in the present paper. The major result is that a **non-negative quasi-option value** exists implying that a decision maker is mistaken if he or she does **not** take into account, in today's decision, the new information emerging in the future. The mistaken decision maker has an unwarranted **anti-preservation bias**. The principal message is that "good decisions" about irreversibilities cannot be reached unless the best possible use of all available information is made. When new information is in the offing the optimal procedure may (but need not always) be a strategy of sequential decision making postponing the decision to destroy the resource characteristic until the point in time in which the new information becomes available.

2. POLLUTION IRREVERSIBILITY UNDER CERTAINTY

Consider an economy in which two outputs are produced: a consumption good (with quantity y), representing the "national product", and a pollutant as an undesired joint product. Labor is the economy's only productive factor. Its endowment, l_0, is assumed constant over time. Labor can be used both for the production of the consumption good and for the reduction of pollutants via intra-industrial abatement. The associated technology can be modeled [Pethig 1979] by the strictly concave production function,

$$y = Y(l_0, m), \tag{1}$$

where $Y(l_0, 0) \geq 0$, $Y_m > 0$, and where $m \in [0, \bar{m}]$ is the amount of pollutants which is produced along with the consumption good and then discharged into the environment. Observe that in function Y the emission **m** is treated as if it were an input (which can be varied with fully employed labor) even though it is clearly an undesired output. The input interpretation is both appropriate and convenient, in fact, because in addition to being an output the emission of that output constitutes the production sector's demand for a productive factor, nature's waste assimilation services.

In order to model the environmental impact of waste discharges, we characterise the environment by its quality, denoted q. Clearly, q has a natural upper bound, q_u, namely the quality of the totally unpolluted environment. There is no lower bound for q, however, so that the initial value of the environmental quality (at time $t = 0$) is some number $q_o \in (-\infty, q_u]$. The environmental quality is assumed to change in time according to

$$\frac{dq}{dt} =: \dot{q} = Q(z), \tag{2}$$

where $Q(0) = 0$, $Q_z < 0$ and where z is the excess demand for (nature's) assimilation services. Restricting the subsequent investigations to the case where the assimilative capacity of the environmental resource (i.e. nature's supply of assimilative services) is zero, means that

$$z = m. \tag{3}$$

Moreover, the function Q is specified by $Q_z(z) = -1$ for z, so that (2) becomes $\dot{q} = m$. (With this specification the function Q is clearly redundant; but it helps to clarify the conceptual issue, in particular, the interpretation of the equations (6) and (7) below).

At every point in time, society's (or a decision maker's) evaluation of consumption, y, and environmental quality, q, is represented by a welfare function. In view of (1) the evaluation of y uniquely determines that of m, so that the welfare function can be expressed as

$$w = W(m, q). \tag{4}$$

The function W is assumed to be strictly increasing strictly concave and separable ($W_{mq} = 0$). The description of our model is completed by introducing the intertemporal objective function

$$\int_o^\infty e^{-\delta t} W(m, q)dt, \tag{5}$$

where δ is a positive and constant rate of (social) time preference. Now the problem of optimal intertemporal pollution control can be stated as maximising (5) subject to (1) - (3). Necessary optimality conditions are, for all $t \in [0, \infty)$,

$$\lambda = W_m Q_z^{-1} = W_m =: MB_m, \tag{6}$$

$$\hat{\lambda} := \frac{\dot{\lambda}}{\lambda} = \delta + \frac{W_q Q_z}{W_m} = \frac{\delta}{MB_m} (MB_m - \frac{MC_m}{\delta}), \tag{7}$$

derived from the Hamiltionian $H = W(m, q) + \lambda Q(z)$. The co-state variable λ is easily interpreted as the (marginal) benefit from emitting an extra unit of pollutants (at time t). According to (7), its relative change, $\hat{\lambda}$, has the same sign as the bracketed term which is equal to the benefit-cost difference of a small additional waste discharge. In (7), $MC_m := - W_q Q_z = W_q$ is the **instantaneous** cost of emitting the last unit of pollutants at some point in time t. But since emission in t causes a **permanent** reduction in environmental quality, we have $MC_{mt'} = MC_{mt}$ for all $t' \geq t$. Consequently, the **overall** marginal cost of emission in t is the present value of all future marginal costs, namely MC_m/δ.

This information is important for characterising the optimal steady state defined by $\dot{m} = \dot{q} = \dot{\lambda} = 0$: The process of irreversible pollution ought to be stopped when the last unit of pollutants emitted into the environment causes a benefit of increased consumption **equal** to the environmental damage associated to that last unit, where it is taken into account that the benefit is only "one moment's bliss", whereas society suffers under the marginal damage perpetually.

Technically speaking, $\dot{\lambda} = 0$ is equivalent to $\delta W_m = W_q$. This equation defines implicitly all tuples (m, q) for which $\dot{\lambda} = 0$ is satisfied. In fact, since we assumed $W_{mq} = 0$, it is possible to describe the $\dot{\lambda} = 0$ locus explicitly with the help of a function F, defined by

$$m = F(q) := W_m^{-1}(\frac{W_q(q)}{\delta}), \tag{8}$$

satisfying $F_q > 0$. Moreover, it is straightforward from (7) that $m \gtreqless F(q) \Leftrightarrow \lambda \lesseqgtr 0$. Finally, differentiation of (6) with respect to time yields $\dot{\lambda} = W_{mm}\dot{m}$. Hence $\dot{\lambda} \lesseqgtr 0 \Leftrightarrow \dot{m} \gtreqless 0$. All this information allows us to characterise the optimal time path of pollution as illustrated in figure 1.

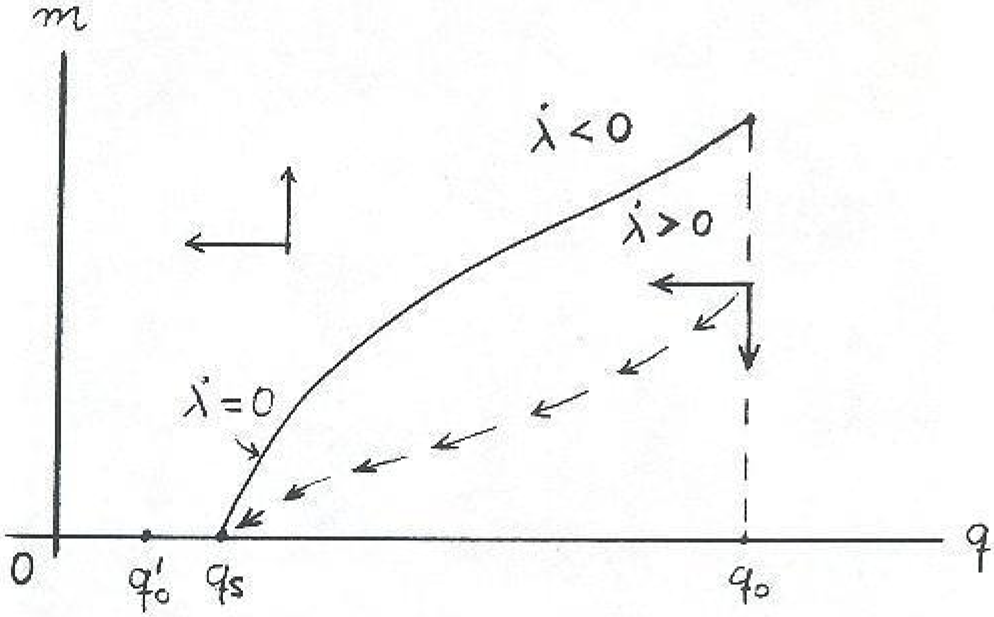

Figure 1: Optimal pollution control under certainty, when destructible resource characteristics are absent

As this diagram shows, if $q_o > q_s$, then it is optimal to approach the steady state quality q_s by gradually diminishing emission and consumption ($\dot{y} = \dot{m}Y_m < 0$). If the initial environmental quality is less than q_s (like q_o' in figure 1), then it is optimal to stay at q_o', that is, not to discharge any (more) waste.

Since $F_q(q) > 0$ for all $q \in (-\infty, q_u]$ we know that if there is a steady state q_s (defined by $F(q_s) = 0$), then it is unique. Inspection of (7) shows that the optimal program implies ever increasing pollution, if and only if $\delta W_m(0) > W_q(q)$ for all $q \in (-\infty, q_u)$. Therefore it is sufficient for an optimal program to converge to a steady state that there exists $\tilde{w} \in \mathbb{R}_{++}$ such that

$$\lim_{m \to 0} W_m(m) = \tilde{w} \quad \text{and} \quad \lim_{q \to -\infty} W_q(q) > \delta \tilde{w}.$$

In fact, $W_q(q_o) \geq \delta W_m(o)$ is sufficient for the optimality of the zero-pollution steady-state strategy $q_s = q_o$.

3. CHARACTERISTICS IRREVERSIBILITY UNDER CERTAINTY

Suppose there is a characteristic r of the environmental resource and a value of environmental quality such that

$$r_t = R(q_t) \begin{cases} = 1, & \text{if } q_z \geq q_r \text{ for all } \tau \leq t, \\ = 0 & \text{otherwise,} \end{cases} \tag{9}$$

i.e. the characteristic disappears as soon as the quality drops below q_r for the very first time, and it remains absent ever after its destruction. The consequences of a switch from $r = 1$ to $r = 0$ may be modeled as shifts in the production function and/or in the evaluation function. For convenience of exposition, we restrict ourselves to shifts in the welfare function such that the function W from (4) is replaced by

$$\tilde{W}(m, q, x) = W(m, q) - x(1 - R(q)). \tag{10}$$

For the purpose of reference, consider first the degenerate case $x = 0$ and assume that in this situation an unique steady state q_s exists such that $q_s < q_r < q_o$. Then environmental quality declines over time from q_o to q_s as illustrated by the curve AD in figure 2. Now replace $x = 0$ in (10) by $x = x_o > 0$. If it is still optimal to destroy the resource characteristic under this modified assumption, then the optimal time path will coincide with that for $x = 0$, because the parametric shift specified in (10) leaves unaffected the marginal properties of the welfare function. Hence the conditions (6) and (7) still hold which determine the optimal time path. But it is conceivable that the preservation strategy BG in figure 2 is superior. More specifically, denote by $d = 1$ and $d = 0$ the decision to preserve and to destroy the resource characteristic, respectively, and let $B(d, x_o)$ be the value of the objective function (5) with W replaced by $\tilde{W}$ from (10). $B(1, x_o)$ means that the preservation strategy ($d = 1$ and expected value of the objective function. Hence the criterion for the optimality of the preservation strategy becomes

$$\Sigma_j p(x_j)\cdot B(1,\ x_j) = B(1,\ x_o) \geq \Sigma_j p(x_j)\cdot B(0,\ x_j), \tag{12}$$

where x_j, $j = 1,\ldots,n$, are the possible realisations of the random variable x which occur with probability $p(x_j) \in [0,\ 1]$, $\Sigma_j p(x_j) = 1$. As the comparison of the equations (11) and (12) shows, the effect of uncertainty simply consists of replacing discounted welfare streams by their expected discounted values.

The preceding arguments implicitly presupposed that with the passing of time nothing further is learnt about the value of x. But there is, of course, always the chance of better information about the benefits of the characteristic to become available in the future implying the potential of a better decision based on that new information provided that in the interim the decision to destroy the characteristic ($d = 0$) has not yet been taken. (In fact, there is even a strong incentive to generate better information while postponing the decision to destroy; but that issue is beyond the scope of the present paper). In what follows, we introduce the simple assumption that the value of x becomes known both unconditionally and with certainty at some future time $\theta > \tau$, where τ is that particular point in time at which the environmental quality reaches the threshold value q_r (from eq. (9)) along the optimal trajectory.

The structure of the problem is easily illustrated with the help of figure 2. The path ABCD represents the optimal program conditional on deletion of the characteristic at time τ. When point C is reached on that path at time θ, the new information is useless in the sense that one cannot reverse the decision to destroy, if the costs of this decision should turn out to be very high. The alternative option to preserve the characteristic at least until time θ means to choose the path ABE during the time interval $\alpha := [\tau,\ \theta]$. This strategy leaves the decision maker with the option of following either EG or EF during the time interval $\beta := (\theta,\ \infty)$ after the new information will have emerged. Obviously, the decisive question is whether or not the resource characteristic should be preserved during the time interval α.

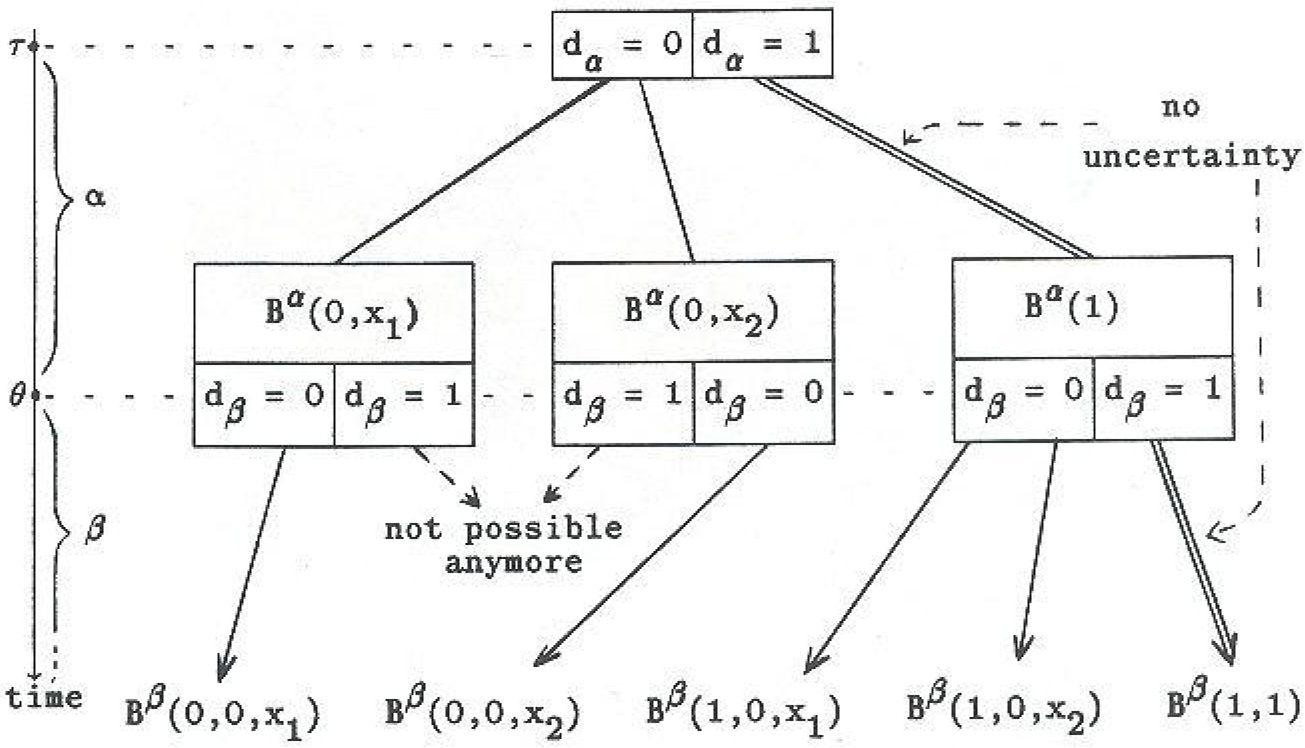

Figure 3: Options of sequential decision making

Figure 3 supplements figure 2 by illustrating the decision maker's options and the values associated to them. For example $B^\beta(1, 0, x_1)$ denotes the value of the decision to destroy the characteristic at time θ ($d_\beta = 0$) when it was preserved until then ($d_\alpha = 1$) and when the random variable takes the value x_1. $B^\beta(1, 0, x_1)$ corresponds to the line segment EF (as does $B^\beta(1, 0, x_2)$, because changes in x do not affect marginal conditions).

The decision maker can solve his or her problem of optimal pollution control either by making use of the future information or by ignoring it. Consider first the latter approach. Disregarding today the new information emerging in θ means that the decision maker copes with uncertainty by resorting to the expected evaluations. More specifically, using the notation of figure 3 the value $V^*(1)$ which he or she places on the decision $d_\alpha = 1$ is

$$V^*(1) := B^\alpha(1) + \max \{B^\beta(1,1),\ \Sigma_j p(x_j) B^\beta(1, 0, x_j)\} \tag{13}$$

The decision $d_\alpha = 0$ has the value

$$V^*(0) := \Sigma_j p(x_j)[B^\alpha(0, x_j) + B^\beta(0, 0, x_j)].$$

Consequently, the decision to preserve the characteristic during the period α is optimal, if and only if

$$V^*(1) \geq V^*(0). \tag{14}$$

Consider now the decision maker's strategy not to ignore the prospect of new information. The value that her or she places on the decision to preserve the characteristic in time interval α (d_α = 1) is then given by

$$\hat{V}(1) := B^\alpha(1) + \Sigma_j p(x_j)\cdot[\max \{B^\beta(1, 0, x_j), B^\beta(1, 1)\}], \tag{15}$$

To the decision $d_\alpha = 0$ he or she attaches the value $\hat{V}(0) = V^*(0)$. It follows that the preservation strategy is considered optimal if and only if

$$\hat{V}(1) \geq V^*(0) \tag{16}$$

The information-regarding approach would be equivalent to the information-ignoring strategy, if it were true that $\hat{V}(1) = V^*(1)$ for arbitrarily distributed random variables x. For the simple case where x has only two realisations, x_1 and x_2, we wish to show that this equality does not necessarily hold. Inspection of (17) shows that the term $\Sigma_j p(x_j)\cdot[\max \{B^\beta(1, 0, x_j), B^\beta(1, 1)\}]$ attains either of the values:

$$\begin{aligned}
h_1 &:= p(x_1)B^\beta(1, 0, x_1) + p(x_2)B^\beta(1, 1),\\
h_2 &:= p(x_1)B^\beta(1, 1) + p(x_2)B^\beta(1, 0, x_2),\\
h_3 &:= p(x_1)B^\beta(1, 1) + p(x_2)B^\beta(1, 1),\\
h_4 &:= p(x_1)B^\beta(1, 0, x_1) + p(x_2)B^\beta(1, 0, x_2).
\end{aligned}$$

We know from the definition of (15) that if $\hat{V}(1) = B^{\alpha}(1) + h_i$, then $h_i \geq h_j$ for all $i, j = 1,\ldots,4$ and $i \neq j$. On the other hand, the term $\max \{B^{\beta}(1, 1), \Sigma_j p(x_j)B^{\beta}(1, 0, x_j)\}$ in (13) takes either the value h_3 or h_4, and it is true that if $V^*(1) = B^{\alpha}(1) + h_i$, then $h_i \geq h_j$ for $i, j = 3, 4, j \neq i$. Consequently, if it turns out that h_i is a maximum element in both sets $\{h_3, h_4\}$ and $\{h_1, h_2, h_3, h_4)$, then $\hat{V}(1) = V^*(1)$. In all other cases one clearly has $\hat{V}(1) > V^*(1)$.

It has been established in the literature [Arrow and Fisher 1974, Henry 1974, Freixas and Laffont 1984, Fisher and Haneman 1986] that the above result holds under much more general assumptions. The difference

$$QOV := \hat{V}(1) - V^*(1)$$

has become known as **quasi-option value** so that another way to state the principal result is, in fact, the observation that the quasi-option value is non-negative.

Some authors like Henry [1974] and Fisher and Hanemann [1986] refer to QOV as the "option value", while others use the term "option value" for a phenomenon distinctly different from QOV. This semantic confusion has its origin in the historical development of these concepts. Fortunately, the substantive issues have been clarified recently, as demonstrated by Bishop [1986] and Freeman [1986]. The option value as defined in these two articles will not be discussed in the present paper, because it "focuses attention on the individual economic agent as s/he evaluates alternatives under uncertainty" [Bishop 1986, p. 147] while for quasi-option value (QOV) the focus ought to be, as it is in our paper, "on the public decision maker who is evaluating public politics or projects under uncertainty" (ibidem). This point is also forcefully made by Freeman [1986].

It should be emphasised that the information-regarding approach is the correct decision-making procedure. Decision makers who ignore the prospect of new information mistakenly tend to underestimate the value of preserving the characteristic during period α. As table 1 spells out, if their **wrong** decision-making procedure leads to an incorrect decision - which need not inevitable be the case - it is always the decision to destroy the characteristic when it should have been preserved during the time interval α.

<table>
<tr><td rowspan="3">Correctness of the decision on d_α
without taking the new information into account
$\Longrightarrow$</td><td>$V^*(1) > V^*(0)$</td><td colspan="2">$V^*(1) < V^*(0)$</td></tr>
<tr><td colspan="2">$\hat{V}(1) > \hat{V}(0)$</td><td>$\hat{V}(1) < \hat{V}(0)$</td></tr>
<tr><td>correct:
$\hat{d}_\alpha = d^*_\alpha = 1$</td><td>incorrect:
$\hat{d}_\alpha = 1$, but
$d^*_\alpha = 0$</td><td>correct:
$\hat{d}_\alpha = d^*_\alpha = 0$</td></tr>
</table>

Table 1: Mistakes in decisions on irreversibilities

A numerical example may help to illustrate this point. Suppose that $p(x_1) = 0.6$, $p(x_2) = 0.4$, $e^{-\delta\theta} = 0.9$ and that $B^\alpha(1) + B^\beta(1, 1)$ and $B^\alpha(0, x_j) + B^\beta(0, 0, x_j)$ have values as shown in table 2:

	x_1	x_2
$B^\alpha(1) + B^\beta(1, 1)$	40	40
$B^\alpha(0, x_j) + B^\beta(0, 0, x_j)$	70	20
$B^\beta(1, 1)$	36	36
$B^\beta(1, 0, x_j)$	63	28

Table 2: An example for incorrect decision making

These data imply the numbers in the third and fourth row of table 2. Furthermore, it is true that $h_1 = 52.2$, $h_2 = 28.8$, $h_3 = 36$, and $h_4 = 45$. Therefore

$$\begin{aligned} V^*(0) &= \hat{V}(0) = 0.6\cdot 70 + 0.4\cdot 20 = 50, \\ V^*(1) &= 4 + \max\{h_3, h_4\} = 49, \\ \hat{V}(1) &= 4 + \max\{h_1, h_2, h_3, h_4\} = 56. \end{aligned}$$

It follows that the mistaken decision maker would opt for destroying the characteristic at time τ ($d_\alpha = 0$), because $V^*(0) - V^*(1) = 1 > 0$. The correct decision is, however, $d_\alpha = 1$, because $\hat{V}(1) - \hat{V}(0) = 6.2 > 0$. In this example, the quasi-option value is $QOV = 7.2$.

The (positive) quasi-option value has been identified as a **value of information**, conditional on retaining the decision leading to an irreversible state [Conrad 1980, Fisher and Hanemann 1986]. It does not follow, however, that the quasi-option value should be considered a separate or additional component of benefit in applied benefit-cost analyses, as some of the earlier literature seemed to suggest. The message of the non-negativity of the quasi-option value is, instead, to avoid mistaken decision making. To the extent that conventional benefit-cost analysis neglected the (present) value of future information [Bishop 1986, p. 150] careful consideration of the information issue will have significant implications for applied research.

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