Paper prepared for presentation at the Econometric Society European Meeting 1999 (section 'Economic Theory') in Santiago de Compostela, August 29 to September 1, 1999

Product Durability, Solid Waste Management, and Market Structure

Marco Runkel*
University of Siegen
Department of Economics IV
Hölderlinstr. 3
D-57068 Siegen
Germany

e-mail: runkel@wap-server.fb5.uni-siegen.de

June 21, 1999

Abstract

For a durable consumption good which turns into waste after consumption, the socially optimal durability increases with an increase in the marginal environmental damage. In a laissez-faire equilibrium under perfect competition, producers fail to provide an efficient product design, i.e. durability is inefficiently small, whereas the amount of solid waste is inefficiently large. The market failure is corrected simply by Pigouvian taxation which also can be interpreted as an extension of the producer responsibility. In the case of imperfect competition (oligopoly or monopoly) Pigouvian taxation indeed ensures an efficient durability but generally not an efficient amount of solid waste.

JEL classification: D40, D62, H23, L68, Q23
key words: product durability, environmental externality, Pigouvian taxation, producer responsibility, market structure

*) I would like to thank Prof. Rüdiger Pethig and Thomas Eichner for very helpful comments. All remaining errors are my own responsibility. Support from the German Research Foundation (DFG) is gratefully acknowledged.

1 Introduction

Previous contributions to the economics of *solid waste management* such as for example DINAN (1993), FULLERTON/KINNAMAN (1995), KOHN (1995) and HIGHFILL/MCASEY (1997) ignored the fact that the process of waste disposal, treatment and recycling and consequently the extent of the accompanying environmental damage is influenced not only by the quantity of the consumption goods but also by several product characteristics. These characteristics generally are chosen by the producers such as to minimize the production cost without recognizing external effects at the end of the product-life-cycle. As a consequence one would expect allocative inefficiency due to the *product design* of consumption goods.

This type of environmental distortion has already received much attention in the political discussion of solid waste management. Nevertheless, the economic literature does not provide a sound theoretical foundation of that issue of product-life-cycle analysis. Among the few exceptions are FULLERTON/WU (1998) who consider the package rate and the recyclability of consumption goods. Assuming that markets fail to be active they suggest possible tax/subsidy schemes to correct the pertinent distortion. In two recent papers, EICHNER/PETHIG (1999a,b) specify the concept of recyclability by explicitly taking into account the recycling sector as well as the material content of the products which is measured by the amount of material per unit of output. With respect to this product characteristic they investigate the ability of markets to provide an efficient resource allocation as well as the possible policy instruments for restoring allocative efficiency in case the markets fail.

By abstracting from the possibility of recycling, the present paper examines a further product characteristic which is important for solid waste management, namely the *durability of consumption goods*. At first glance, the relationship between the product durability and the waste flow of an economy may not be easy to see. In order to clarify this point, consider the following simple example. Suppose a producer supplies a car with a lifetime of 10 years or, alternatively, two cars with a lifetime of 5 years each. If the material needed for a car with a 5-year lifetime is more than half the weight of the car with a 10-year lifetime, then the amount of solid waste per year is greater in the case of 5-year cars than in the case of 10-year cars. Since this hypothesis regarding the comparative weight of 5-year and 10-year cars is plausible, an increasing product durability tends to reduce the material throughput of the economy and thus the amount of waste per period as well as the accompanying environmental damage.

To the best of our knowledge, WAGNER (1992) is the only author who explicitly models the waste flow of durable goods. But he doesn't include environmental externalities into his model and so obtains the - not very surprising - result that the durability of consumption goods is Pareto efficient. If there are no markets for product characteristics and environmental quality, however, producers in a laissez-faire economy generally won't internalize the external cost and thus they fail to provide an efficient product design. Therefore, the present paper reexamines the issue of durability choice under the environmental perspective. A dynamic model of a durable consumption good is developed. The built-in product durability is a decision variable of the producers where an increase in the durability leads to an increase in the production cost. After consumption the used products are scrapped and cause environmental damage through for example waste transport, incineration or landfilling. It is shown that price-taking producers in a laissez-faire economy choose an inefficiently low durability and an inefficiently large material throughput with an accompanying inefficiently large amount of solid waste. Furthermore, it turns out that, besides more complex tax/subsidy schemes, simply a Pigouvian tax on waste is sufficient to ensure an efficient resource allocation. In our framework, as will be argued, the Pigouvian taxation can also be interpreted as an extension of the producer responsibility which means that the producer legally remains the owner of the products and at the end of the product-life-cycle is forced to pay the disposal costs. The extension of the producer responsibility has received much attention in the political discussion of solid waste management because it is expected to provide incentives for an efficient product design (OECD (1998)). Our analysis will support this view with respect to an efficient regulation of the product durability.

The choice of product durability is not a new issue in economic theory. By completely ignoring the environmental perspective, there has been a long debate about the relationship between the *market structure* and the durability of consumption goods. According to the so-called SWAN'S independence result (SWAN (1970), SIEPER/SWAN (1973)) the product durability under monopoly equals the one under perfect competition. Despite the fact that it has been shown to be sensitive to some of SWAN's original assumptions (for a survey see SCHMALENSEE (1979) as well as the introduction in the more recent article of MUELLER/PERES (1990)), the independence result is expected to hold also in the case of an oligopoly (GOERING (1992)). At first glance, this seems to be a useful result in view of the present paper because one may concentrate on the environmental impact of the product durability under one market structure and neglect the other ones. Of course, this is not true since the material throughput of an economy also is affected by the quantity of the durable goods. Under imperfect competition this quantity usually differs from that under perfect competition due to some market power of the producers. Hence, the amount of solid waste and the accompanying environmental damage depend on the market structure even when the durability is the same. The present paper additionally aims to outline the differences in the optimal waste management policies under alternative market structures. It is shown that under imperfect competition (monopoly as well as oligopoly) the Pigouvian tax on waste (or the extension of the producer responsibility) indeed provides the right incentive for an efficient product design but generally is not capable of fully correcting the distortion with respect to the quantity of the consumption goods. Therefore, the amount of solid waste is inefficient even when the durability is on its socially optimal level. To restore the overall efficiency, a subsidy on the firm's stock of the durable good is also required.

The plan of the paper is as follows. In section 2 the model is outlined and the socially optimal outcome is described. Section 3 contains the market solution where we distinguish the case of free entry from the case of entry barriers. Under free entry, as will be seen, the market structure of perfect competition is reached while entry barriers lead to an oligopoly. The latter case is too complex to be analysed in the general model. Thus we restrict ourself to numerical simulations. Section 4 returns to the more general model and considers a multi-plant monopoly. Section 5 offers a summary and conclusion.

2 Welfare Maximizing Plan

2.1 General Surplus and Cost Functions

The following analysis adopts the framework of KAMIEN/SCHWARTZ (1974) and SWAN (1977) and extends their approach by explicitly taking into account a flow pollution caused by the scrapped units of durable goods. The supply side of the economy is represented by n identical firms where in the present paper two cases are distinguished. In the first case, the number of firms is treated as an endogenous variable. Hence, in the problem of welfare maximization n is explicitly chosen by the social planner whereas in the analysis of markets n characterizes the market structure and is determined by the number of firms which enter the market, namely those that have nonnegative profits. In the second case, the number of firms will be taken as given for example due to political or legal constraints or due to prohibitively large entry cost. In this case the market structure is fixed. For the moment, however, suppose an endogenous number of firms.

Each firm produces a durable consumption good with the cost function $K(y, s, \delta)$. $y(t)$ is the production rate in period t and s is the plant size. The durable good is modeled as a perfectly divisible good which "evaporates" at the built-in decay rate δ, i.e. at every point in time a fraction δ of the durable's stock is scrapped. The term $1/\delta$ can be interpreted as the average built-in durability of the consumption good. Assume that an increase in the production rate or a decrease in the decay rate (an increase in the durability) leads to an increase in the production cost at increasing rates, i.e. $K_y, K_{yy} > 0, K_\delta < 0, K_{\delta\delta} > 0$. With respect to plant size the cost function exhibits an U-shape.[1] The production rate is determined for every t while the plant size and the durability are chosen once-and-for-all. This rather strong assumption can be rationalized by prohibitively large adjustment costs for variations of s and δ after $t = 0$.

Every firm rents rather than sells its output. The firm remains the owner of the produced units, accumulates these units over time and sells the services of the durable good rather than the good itself. The firm's accumulated stock $c(t)$ of the durable good for time t is enlarged by the produced units in t and lowered by the scrapped units in t. Hence, $c(t)$ satisfies the differential equation[2]

$$\dot{c}(t) = y(t) - \delta c(t) \qquad \text{with} \qquad c(0) = 0. \tag{1}$$

As SWAN (1970), p. 886 argues, the strategies of renting and selling the products are equivalent when the consumers are rational. Under this assumption the sales price of

[1] For a further discussion of these properties see the specification of K on p. 7.

[2] The initial stock must be zero because otherwise there would be units of the durable good produced prior to the decision period with a built-in durability not necessarily equal to the one chosen for $t \geq 0$.

an unit of the durable good equals the present value of its rentals and thus the firm's revenue and decisions are the same for the strategy of renting and the strategy of selling the good. Assuming the renting strategy in the present paper simplifies the formulation and solution of the model. However, when interpreting our results we will also refer to the selling strategy in some cases.

The demand side of the model is represented by the demand function $P(nc)$. This function gives the rental price for an unit of the consumption good (or the price for an unit of the durable's service) which depends on the total industry stock nc. An increasing stock causes a decreasing rental price, i.e. $P' < 0$. The consumer surplus is defined as $S(nc) = \int_0^{nc} P(C)dC$ with $S'(nc) = P(nc) > 0$ and $S''(nc) = P'(nc) < 0$. Hence, an increasing stock leads to an increasing surplus at decreasing rates. In addition to this positive effect, a negative effect on consumers is introduced in form of a flow pollution: In every period t, the quantity $n\delta c(t)$ of the industry stock which is scrapped turns into solid waste and causes an externality through for example waste transport, incineration or landfilling. The environmental damage is denoted by $D(n\delta c)$ with $D' > 0$ and $D'' > 0$, so an increasing scrapping rate leads to increasing environmental cost at increasing rates. This treatment of solid waste in a dynamic framework does not seem to be appropriate for all pollution problems because in some cases it is the stock of the solid waste which causes environmental damage. However, the dynamic framework is not chosen to focus on the intertemporal effects of waste accumulation but primarily to model durable consumption goods. Thus, as an approximation it is suitable to neglect dynamic effects of the waste stock to keep the model as simple as possible.[3]

To obtain the socially optimal outcome in the economy just described, consider a social planner who maximizes the social welfare, i.e. who solves the problem[4]

$$\max_{y(t),s,\delta,n} \int_0^{\infty} e^{-\rho t}\Big(S(nc) - nK(y,s,\delta) - D(n\delta c)\Big)dt \qquad (2)$$

subject to (1) where $\rho > 0$ stands for the social discount rate. The social welfare equals the consumer surplus net of the production cost and the environmental cost. For time t, the instantaneous welfare is represented by the terms in brackets in (2). To solve the problem (2) the social planner chooses an entire time path of the production rate as well as the once-and-for-all values for the plant size, the durability of goods and the number of firms in order to maximize the discounted social welfare over an infinite planning horizon. For the time being we do not impose nonnegativity constraints. Rather, conditions under which the optimal production is positive are provided below.

Imagine the social planner to proceed step by step. She first keeps the plant size, the durability and the number of firms constant and determines the whole path of the production rate and the accompanying path of the durable's stock. By inserting these

[3] In the taxonomy of TIETENBERG (1988), p. 307 we consider the *fund* pollutants and ignore the *stock* pollutants of the waste flow. However, it is not expected that the main results of our analysis change if the waste stock is explicitly taken into account.

[4] We suppress the time variable t so far as misunderstandings are not possible. Subscripts denote partial derivatives. The derivative of a function with only one argument is written with a prime.

welfare-maximizing time paths into the objective functional she obtains a welfare function which depends on the plant size, on the durability and on the number of firms and therefore, in the second step, can be used to determine the socially optimal values of these variables. In the first step, for given s, δ and n, equation (2) becomes a problem of optimal control with the present-value Hamiltonian

$$\mathcal{H} = e^{-\rho t}\Big(S(nc) - nK(y,s,\delta) - D(n\delta c)\Big) + \mu\Big(y - \delta c\Big)$$

where μ is the costate variable. Since $S - nK - D$ is concave in y and c the first-order conditions

$$\mathcal{H}_y = -ne^{-\rho t}K_y(y,s,\delta) + \mu = 0, \qquad \dot{\mu} = -\mathcal{H}_c = -ne^{-\rho t}\Big(S'(nc) - \delta D'(n\delta c)\Big) + \delta\mu \quad (3)$$

are necessary and sufficient to solve the problem. The second equation in (3) is solved for μ with the help of the transversality condition $\lim_{t\to\infty}\mu(t) = 0$ and the method of variation of the parameter. The result is

$$\mu(t) = n\int_t^\infty e^{-\rho v}e^{-\delta(v-t)}\Big(S'(nc(v)) - \delta D'(n\delta c(v))\Big)dv. \quad (4)$$

The last two factors of the integrand in this equation stand for the marginal change in the net consumer welfare (consumer surplus less environmental cost) in period v due to a marginal change in the firm's stock of the durable in period t. Discounting this marginal value to period 0, integrating from t to infinity and summing up over all firms yields the total social value of an additional unit of the firm's stock in period t. Thus, μ can be interpreted as the social shadow price of c.

Combining (4) with the first equation in (3) yields

$$K_y(y(t),s,\delta) + \int_t^\infty e^{-(\rho+\delta)(v-t)}\delta D'(n\delta c(v))dv = \int_t^\infty e^{-(\rho+\delta)(v-t)}S'(nc(v))dv.$$

The LHS equals the marginal social cost (marginal production cost plus marginal environmental cost) and the RHS equals the marginal social benefit (marginal consumer surplus) of a marginal increase in the output of period t, all in current values. When both sides are differentiated with respect to time, we obtain the differential equation

$$\dot{y} = \frac{(\rho+\delta)K_y(y,s,\delta) - S'(nc) + \delta D'(n\delta c)}{K_{yy}(y,s,\delta)}. \quad (5)$$

(1) and (5) are a system of differential equations whose solution represents the socially optimal production rate $y^o(t;s,\delta,n)$ and the socially optimal stock $c^o(t;s,\delta,n)$ of the durable. This solution depends on the plant size, the product durability and on the number of firms. From (1) and (5) we easily obtain $(\partial y/\partial c)|_{\dot{c}=0} > 0$, $\partial\dot{c}/\partial c < 0$, $(\partial y/\partial c)|_{\dot{y}=0} < 0$ and $\partial\dot{y}/\partial c > 0$. Hence, the phase diagram in figure 1 characterizes the qualitative properties of the socially optimal time path. The steady state (c^*, y^*) is a saddle point. The arrowed line represents the optimal trajectory. Since the initial stock of the durable good is assumed to be zero, the initial value of the production rate must be positive. The path

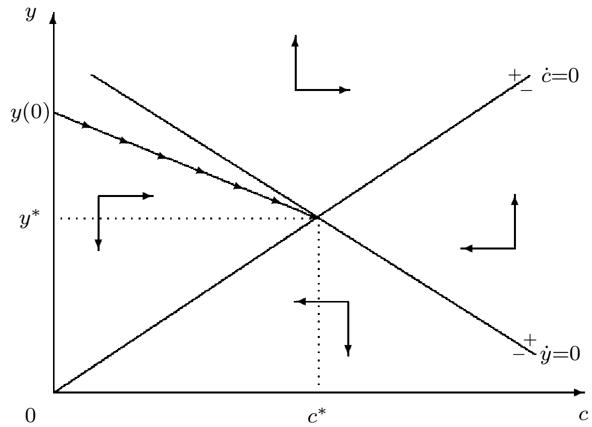

Figure 1: Socially optimal time path

of the production rate decreases monotonely until it reaches its steady state value. The socially optimal stock increases monotonely from zero to its long-run level.

After having calculated the efficient production rate, the social planner now turns to determining the socially optimal values for the plant size, the durability and the number of firms. For that purpose, she has to calculate the optimal production rate and the accompanying stock of the durable good from the above differential equations, insert them into the objective functional and differentiate this with respect to the three variables. However, in the general case it is not possible to explicitly specify the solution to the above system of differential equations. But qualitative insights are obtained by using the dynamic envelope theorem proposed by LaFrance/Barney (1991). By concentrating only on the socially optimal value of the product durability, we get the necessary condition $\int_0^\infty \mathcal{H}_\delta|_{(y^o,c^o)}\, dt = 0$ or

$$\int_0^\infty e^{-\rho t} K_\delta(y^o, s, \delta)dt + \int_0^\infty e^{-\rho t} c^o K_y(y^o, s, \delta)dt + \int_0^\infty e^{-\rho t} c^o D'(n\delta c^o)dt = 0. \qquad (6)$$

A marginal decrease in the product durability has three effects. First, for every time t it brings about a (direct) marginal decrease in the production cost whose present value equals the first integral in (6). Second, it also leads to an (indirect) marginal increase in the production cost, because in order to maintain the stock of the durable good an increasing production rate is required for every time t. The present value of these additional costs is captured by the second integral in (6). Third, a decreasing durability causes an increasing environmental damage because the amount of solid waste for every time t increases. The present value of this effect equals the third integral in (6). Hence, roughly speaking, condition (6) says that the product durability is socially optimal if the saved production costs due to a decrease in durability are just offset by the cost of added production to maintain the stock of the durable and the additional environmental cost.

2.2 Quadratic Surplus and Cost Functions

To compare the efficient durability with the one arising under different market structures, a parametric specification of the demand and the cost functions is warranted. Following Kamien/Schwartz (1974), p. 294 we choose second-order approximations. More specifically, consider the linear demand function $P(nc) = \alpha - \beta nc$ with $\alpha, \beta > 0$ and the associated quadratic consumer surplus $S(nc) = \alpha nc - \beta(nc)^2/2$ with $S'(\cdot) = \alpha - \beta nc$ and $S''(\cdot) = -\beta$. In addition, we specify the environmental damage function as $D(n\delta c) = \gamma n\delta c + \varepsilon(n\delta c)^2$ with $\varepsilon, \gamma > 0$ which implies that the marginal environmental costs of the waste flow are positive ($D'(\cdot) = \gamma + 2\varepsilon n\delta c$) and increasing ($D''(\cdot) = 2\varepsilon$).

For the production cost we introduce the quadratic function $K(y, s, \delta) = y^2 + (M(\delta) - 2s)y + s^2$ with $M(\delta) - 2s > 0$ and therefore $K_y(\cdot) = 2y + M(\delta) - 2s > 0$ and $K_{yy}(\cdot) = 2 > 0$. At first glance, this construction of the production cost seems somewhat artificial, but as KAMIEN/SCHWARTZ (1974), p. 294 pointed out the cost function exhibits constant returns to scale in the long-run (defined as the situation in which the plant size minimizes the cost function, i.e. $s = y$) where the long-run unit costs are $M(\delta)$. In the short-run the unit cost function is U-shaped and tangent to the long-run unit cost function at $y = s$. Moreover, as will be seen in section 3, the inclusion of the plant-size s ensures that at the margin (when the number of firms becomes infinite) the model is consistent with previous models on product durability but, at the same time, is more general than these models because it allows the investigation of the case in which the number of firms is finite.

The properties of the function M play a key role in the subsequent analysis. We assume

$$M'(\delta) < 0, \qquad 2M'(\delta) + (\rho + \delta)M''(\delta) > 0. \tag{7}$$

The first condition postulates a positive correlation between the durability and the production cost. The second condition can be rearranged to $-\delta M''/M' > 2\delta/(\rho + \delta)$ which means that the function M is sufficiently convex, or more economically speaking, that a decrease in the durability leads to a relatively great decrease in the long-run unit cost. This condition turns out to be sufficient for the second-order condition of welfare-maximization and *necessary* for the second-order condition of profit-maximization (see lemma 1, 2 and 3). Therefore, it can't easily be discarded.

With the specification of the demand and the cost functions, the welfare-maximizing production rate and stock of the durable good are the solution to an inhomogeneous system of linear differential equations. The system consists of (1) and

$$\dot{y} = (\rho + \delta)y + [\beta n + 2\varepsilon\delta^2 n]c/2 - [\alpha - (\rho + \delta)(M(\delta) - 2s) - \gamma\delta]/2. \tag{8}$$

Equation (8) is obtained by inserting the parametric functions specified above into (5). The solution to this system of differential equations is (see appendix A1)

$$y^o(t; s, \delta, n) = c^* \left[\delta + (\lambda - \delta)e^{-\lambda t}\right], \qquad c^o(t; s, \delta, n) = c^* \left[1 - e^{-\lambda t}\right] \tag{9}$$

with

$$c^* := \frac{\alpha - (\rho + \delta)(M(\delta) - 2s) - \gamma\delta}{2\lambda(\rho + \lambda)}, \qquad \lambda := \frac{\sqrt{(\rho + 2\delta)^2 + 2\beta n + 4\varepsilon\delta^2 n} - \rho}{2}. \tag{10}$$

c^* is the steady state value of the firm's stock of the durable good and $-\lambda$ is the (relevant) root of the characteristic equation. Note, that $\lambda > \delta$. To compute the efficient values of the plant size, the product durability and the number of firms, insert the solution (9) for the production rate and the stock of the durable into the objective functional in (2). This yields the maximum welfare[5]

$$\widetilde{V}(s,\delta,n) = \frac{n}{\rho}\Big(c^{*2}\lambda^2 - s^2\Big).$$

First, we calculate the optimal value of the plant size taking as given δ and n. From (10) we obtain $\partial c^*/\partial s = (\rho+\delta)/\lambda(\rho+\lambda)$. Thus, the necessary condition $\widetilde{V}_s = 2n[c^*\lambda(\rho+\delta)/(\rho+\lambda) - s]/\rho = 0$ yields the socially optimal plant size

$$s^o = \frac{[\alpha - (\rho+\delta)M(\delta) - \gamma\delta](\rho+\delta)}{2\,[(\rho+\lambda)^2 - (\rho+\delta)^2]} \tag{11}$$

where we have used the definition of c^* from (10). The sufficient condition $\widetilde{V}_{ss} = 2n[(\rho+\delta)^2/(\rho+\lambda)^2 - 1]/\rho < 0$ is satisfied due to $\lambda > \delta$. The steady state value of the stock of the durable good can now be recalculated from (10) and (11) as

$$c^* = \frac{[\alpha - (\rho+\delta)M(\delta) - \gamma\delta]\,(\rho+\lambda)}{2\lambda\,[(\rho+\lambda)^2 - (\rho+\delta)^2]} \tag{12}$$

to obtain the maximum welfare as a function of the durability and the number of firms

$$V(\delta,n) := \widetilde{V}(s^o,\delta,n) = \frac{n}{4\rho}\,\frac{[\alpha - (\rho+\delta)M(\delta) - \gamma\delta]^2}{(\rho+\lambda)^2 - (\rho+\delta)^2}. \tag{13}$$

To focus on an economically meaningful solution, we assume

$$1 < (\alpha - \gamma\delta)/(\rho+\delta)M(\delta) < (\rho+\lambda)^2/(\rho+\delta)^2. \tag{14}$$

The first inequality renders the steady state stock c^* and the plant size s^o positive. The second inequality ensures $M(\delta) - 2s^o$ to be positive as assumed in the specification of the model. Some remarks on the assumption (14) are in order. First, it is not redundant, since $\lambda > \delta$ implies the last term of the inequalities to be greater than one. Second, it appears to be more restrictive than it really is: Except for the numerical computations (in which the nonnegativity constraints are explicitly checked), below only the case in which the number of firms grows without bounds will be considered. But the last term in (14) tends to infinity as $n \to \infty$ and the second inequality is satisfied so long as α is finite. Thus, roughly speaking, the above assumption means that the prohibitive price α for the services of durables is finite but large enough for the optimal production to be positive. Finally, the maximum welfare (13) is nonnegative independent of (14).

The socially optimal values of the durability and the number of firms can now be specified with the help of the maximum welfare (13). To avoid notational cluttering, define $A(\delta) := \alpha - (\rho+\delta)M(\delta) - \gamma\delta$ with $A'(\delta) = -[(\rho+\delta)M'(\delta) + M(\delta) + \gamma]$, $A''(\delta) =$

[5] We could avoid this calculation by using the envelope theorem as in the general case. But, as will be seen in the next section, this theorem can't be used to get the solution for a competitive market. To keep the results comparable, proceed as above at the cost of a little more calculation expense.

$-[2M'(\delta)+(\rho+\delta)M''(\delta)]$ and $B(\delta,n) := (\rho+\lambda)^2-(\rho+\delta)^2$. The maximum welfare then becomes $V(\delta,n) = nA^2(\delta)/4\rho B(\delta,n)$. In order to specify the social optimum we need the first and second-order condition for durability,

$$V_\delta(\delta,n)=0 \qquad \Leftrightarrow \qquad -A'(\delta)+\frac{A(\delta)B_\delta(\delta,n)}{2B(\delta,n)}=0 \tag{15}$$

$$V_{\delta\delta}(\delta,n)<0 \qquad \Leftrightarrow \qquad A''(\delta)<\frac{A(\delta)B_{\delta\delta}(\delta,n)}{2B(\delta,n)}-\frac{A'^2(\delta)}{A(\delta)} \tag{16}$$

as well as the first derivative of the maximum welfare with respect to n

$$V_n(\delta,n)=\frac{A^2(\delta)}{2(\rho+2\lambda)[4\delta^2+n(\beta+2\varepsilon\delta^2)+10\rho\delta+11\rho^2+4\delta\lambda+6\rho\lambda]}. \tag{17}$$

Note, that in order to derive (16) we have used (15) and $A>0$ due to assumption (14) as well as $B>0$ due to $\lambda>\delta$. The following lemma is established with the help of (15) to (17).

Lemma 1 (Social Optimum) *If the number of firms is endogenous, then the social optimum has the following properties:*

(i) The number of firms tends to infinity and the steady state is reached immediately.

(ii) The firm's plant size s^o, production rate $y^o(t)$ and stock of the durable good $c^o(t)$ converge to zero for all t.

(iii) The industry stock of the durable good becomes

$$C^o := \lim_{n\to\infty} nc^o(t) = \frac{\alpha-(\rho+\delta^o)M(\delta^o)-\gamma\delta^o}{\beta+2\varepsilon(\delta^o)^2} \qquad \textit{for all } t. \tag{18}$$

(iv) The optimal product durability $1/\delta^o$ is implicitly defined by

$$(\rho+\delta^o)M'(\delta^o)+M(\delta^o)+\gamma+2\varepsilon\delta^o C^o=0. \tag{19}$$

This condition is necessary and sufficient for the socially optimal durability.

Proof: Note first, that for given durability equation (17) implies $V_n>0$ for all finite n. Hence, the socially optimal number of firms is infinite. Since this implies $\lambda\to\infty$ owing to (10) and $y^o(t)\to y^*=\delta c^*$ and $c^o(t)\to c^*$ for all t owing to (9) we have proved *(i)*. The firm's plant size s^o and the steady state stock c^* of the durable converge to zero owing to (11) and (12).[6] This implies $y^o(t), c^o(t)\to 0$ for all t and *(ii)* of lemma 1 is proved. *(iii)* is directly shown by computing the limit value for $nc^o(t)=nc^*$ as $n\to\infty$. To show that *(iv)* is true, let $n\to\infty$ in (15). A and its derivatives are independent of n and therefore remain unchanged. However, $B_\delta/2B \to 2\varepsilon\delta/(\beta+2\varepsilon\delta^2)$ and $B_{\delta\delta}/2B\to 2\varepsilon/(\beta+2\varepsilon\delta^2)$ as

[6] This and the subsequent limit values have been calculated by the author with the help of the Mathematica package. The author hopes that the reader shares the trust in the computational capacity of this package. Details on the calculation can be obtained upon request.

$n \to \infty$. Inserting the first of these limits into the first-order condition (15) and using C^o from (18) proves (19) as a necessary condition for the socially optimal durability. By using the second of these limits in (16) the second-order condition $V_{\delta\delta} < 0$ becomes

$$-[2M'(\delta^o) + (\rho+\delta^o)M''(\delta^o)] < 2\varepsilon\, C^o - \frac{[(\rho+\delta^o)M'(\delta^o) + M(\delta^o) + \gamma]^2}{\alpha - (\rho+\delta^o)M(\delta^o) - \gamma\delta^o} \overset{(18),(19)}{=} \frac{2\varepsilon\beta C^o}{\beta + 2\varepsilon(\delta^o)^2}$$

This inequality is satisfied because (7) renders the LHS negative whereas the RHS is positive due to (14). Hence, the second-order condition is satisfied and (19) is necessary and sufficient for determining the welfare maximizing product durability. ■

There are three remarks on lemma 1. First, although the socially optimal value of the firm's stock of the durable good converges to zero as the number of firms grows without bounds, assumption (14) ensures the socially optimal industry stock of the durable good in (18) to be positive. Second, the optimal time path in the phase diagram of figure 1 becomes vertical since the steady state is reached immediately. In other words, the steady state industry stock of the durable is already produced at the first point in time and prevails thereafter. Correspondingly, the durable production is such that the quantity of scrapped durables is exactly replaced at each point in time. Third, equation (19) has the same marginal cost/marginal benefit interpretation for the durability as the general condition (6): The first term in (19) can be interpreted as the marginal cost saving due to a decreasing durability. M stands for the cost of added production to keep the stock of the durable good constant. The last two terms exactly match the marginal environmental cost in the social optimum. To see this, note that with the help of (18) the marginal environmental damage in the social optimum becomes MD $:= \lim_{n\to\infty} D'(n\delta^o c^o(t)) = \gamma + 2\varepsilon\delta^o C^o$ which equals the last two terms on the LHS of (19). Hence, according to this equation the product durability is socially optimal if the positive effect of a decrease in the durability, namely the saved direct production cost, is offset by the negative effects consisting of the additional production cost to maintain the stock of the durable at its efficient level and the additional environmental cost.

By totally differentiating (18) and (19) the following comparative dynamic results for the product durability, the industry stock of the durable good and the amount of solid waste in the social optimum are obtained (see appendix A2).

	$\partial\alpha$	$\partial\beta$	$\partial\rho$	$\partial\gamma$	$\partial\varepsilon$
$\partial\delta^o$	< 0	> 0	> 0	< 0	< 0
∂C^o	> 0	< 0	< 0	< 0	< 0
$\partial(\delta^o C^o)$	> 0	< 0	$\gtreqless 0$	< 0	< 0

Table 1: Comparative dynamic results of the social optimum

The partial derivatives are all intuitively plausible: An expanding market for the durable good, caused either by an increase in the maximum demand price α or by an increase in the slope $-\beta$ of the demand function, leads to an increase in the efficient durability of consumption goods. To understand this, note that the expanding market leads to an increasing industry stock of durables and therefore c.p. to an increasing amount of solid waste. To partially compensate for this increase in the amount of waste and the accompanying increase in the environmental cost, the social planner has to reduce the decay rate or, equivalently, raise the product durability. An increase in the social discount rate reduces both the socially optimal durability and the socially optimal stock of the durable because the welfare of future generations receives a lower weight in the welfare function. However, the change in the waste flow is ambigous in sign. The most important result of the comparative analysis is captured in the last two column of table 1.

Proposition 1 *An increase in the marginal environmental damage, reflected by an increase in either γ or ε, brings about an increase in the socially optimal durability, a decrease in the socially optimal industry stock of the durable good and a decrease in the socially optimal amount of solid waste.*

Since for a greater γ or a greater ε every scrapped unit causes greater environmental damage, the socially optimal amount of waste must be reduced by lowering the industry stock and by increasing the durability of the consumption goods. This result provides the new insight that the product durability is affected by the extent of the environmental damage caused by the solid waste of durable consumption goods. The existing literature on the product durability (see the articles already referred to in the introduction) implicitly assumes $\gamma = \varepsilon = 0$. But proposition 1 shows that the socially optimal durability increases when γ or ε are increased to a number greater than zero.

Until now we have investigated the case in which the number of firms is endogenously determined by the social planner. As mentioned at the beginning of this section, the case of a fixed number of firms is also of interest in the present paper. When n is fixed and finite, then equation (17) becomes superfluous and the steady state is not reached immediately. The socially optimal values of the production rate, the stock of the durable, the plant size and the durability are determined by (9) to (12), (15) and (16). However, because these equations are far too complex to be solved parametrically, it is quite difficult (if not impossible) to provide useful interpretations and to ascertain the influence the parameters exert on the optimal values. Hence, for the moment we will refrain from this sophistical work. Nevertheless, in section 3.3 some numerical results are provided.

3 Market Solution

3.1 Open-Loop Nash Equilibrium

Now consider the case in which the durable is supplied by n identical profit-maximizing firms. The single firm has to choose the time path of the production rate and the stock of the durable good on the one hand and the plant size and the product durability on the other hand. For constant plant size and durability, the first decision is modeled as a non-cooperative differential game with an open-loop Nash equilibrium:[7] The firm chooses

7) For an introduction to equilibrium concepts in differential games see for example Petit (1990), pp. 207. A recent application of the open-loop Nash equilibrium in a model similar to the present one but with a stock pollution and a nondurable good is given by Benchekroun/van Long (1998).

a time path for its production rate according to what is the best response to the time paths chosen by its competitors. A Nash equilibrium is reached if, for given actions of the other firms, no firm has an incentive to change its own plan. The information structure of the firm's decision is open-loop which means that the firm at time t only recalls the initial state, i.e. the initial value of the durable's stock. Owing to this information structure we can also say that the firm determines the entire time path of the production rate right at the beginning of the planning period. Of course, a firm may consider to deviate from its initial plan as time goes by, but it will not do so because it is known from literature that the open-loop Nash equilibrium is time consistent, i.e. along the equilibrium path no firm is able to make itself better off by changing its original plan (Karp/Newbery (1993), pp. 889). Due to the assumption of perfect foresight the individual firm anticipates the open-loop Nash equilibrium and inserts the equilibrium solution into the objective functional. This yields a profit function which depends on the plant size as well as on the durability and which can be used in the second stage of planning (also right at the beginning of the planning period) to determine the profit-maximizing values of these variables.

To be more specific, consider a single firm with the revenue $R(c) = cP(c + \widetilde{C})$ where $\widetilde{C}$ is the durable's stock of all other firms. The firm believes $\widetilde{C}$ to be independent of its own actions. For the marginal revenue suppose $R'(c) = P(c + \widetilde{C}) + cP'(c + \widetilde{C}) > 0$ and $R''(c) = 2P'(c + \widetilde{C}) + cP''(c + \widetilde{C}) < 0$. The firm solves the problem

$$\max_{y(t),s,\delta} \int_0^\infty e^{-\rho t}\Big(cP(c + \widetilde{C}) - K(y, s, \delta) - \tau_y y - (\tau_w \delta + \tau_c)\, c\Big) dt \tag{20}$$

subject to (1) where it is assumed that the firm's discount rate coincides with the social one. The instantaneous profit equals revenue net of production cost and tax payments. Since in the presence of external cost markets are expected to fail in a laissez-faire economy, the policy maker has introduced a tax τ_y on the firm's output, a tax τ_c on the firm's stock of the durable good and a tax τ_w on the amount of solid waste δc. For the moment, τ_w is simply interpreted as a tax rate. As will be seen in the interpretation of proposition 3, however, this tax captures a wider class of policy instruments. All tax rates are announced by the policy maker right at the beginning of the planning period and are taken as given by the firms. Moreover, the tax rates are assumed to be constant over time. This assumption can be justified by political or legal constraints. It excludes dynamically inconsistent behaviour of the policy maker.

For given plant size and durability, the open-loop Nash equilibrium can be derived by an optimal control approach similar to that already used in section 2: After identifying the firm's shadow price of the durable's stock and the marginal cost/marginal revenue condition for the firm's output we obtain the differential equation[8]

$$\dot{y} = \frac{(\rho + \delta)K_y(y, s, \delta) + (\rho + \delta)\tau_y - P(c + \widetilde{C}) - cP'(c + \widetilde{C}) + \tau_w \delta + \tau_c}{K_{yy}(y, s, \delta)}. \tag{21}$$

(1) and (21) determine a time path of the production rate and of the stock of the durable good which is the firm's best response to $\widetilde{C}$, i.e. to the stock of the durable good chosen

[8] For notational convenience, the same symbols are used here as in the welfare maximization problem, even though their meaning is not the same in general.

by the firm's competitors. If, for given actions of the other firms, no firm has an incentive to deviate from its time path, then an open-loop Nash equilibrium is reached. To keep the analysis tractable we restrict our attention to the symmetric equilibrium. Hence, the equilibrium time path is obtained by inserting the condition $\widetilde{C} = (n-1)c$ or $c + \widetilde{C} = nc$ in (1) and (21) and solving this system of differential equations for the production rate and the stock of the durable. The accompanying phase diagram is analogous to that in figure 1: The production rate decreases until it reaches its steady-state value while the stock of the durable increases from zero up to its long run level.

Now turn to the choice of the plant size and the product durability. Due to the assumption of perfect foresight the individual firm anticipates the symmetric equilibrium and inserts the equilibrium solution right at the beginning of the planning period into the objective functional (20). Since the equilibrium solution depends on the plant size and on the durability, the firm obtains the maximum profit as a function of these variables. To determine the profit-maximizing values of the plant size and the durability, the partial derivatives of the profit function are set equal to zero. It is important to note that even in the general case it is not feasible to use the dynamic envelope theorem as in the welfare maximization because this theorem doesn't apply to differential games. Therefore, we proceed immediately with parameterizing the model. With the same specification of the consumer demand and the production cost as in section 2, the firm's revenue reads $R(c) = \alpha c - \beta c(c + \widetilde{C})$ with $R'(c) = \alpha - 2\beta c - \beta\widetilde{C}$ and $R''(c) = -2\beta$. Next we insert the first derivative of the revenue function together with the equilibrium condition $\widetilde{C} = (n-1)c$ and the derivatives of the production cost function into (21) to get an inhomogeneous system of linear differential equations which consists of (1) and

$$\dot{y} = (\rho + \delta)y + \beta(n+1)c/2 - [\alpha - (\rho + \delta)(M(\delta) - 2s + \tau_y) - \tau_w\delta - \tau_c]/2. \tag{22}$$

The solution to this system, $y^c(t; s, \delta)$ and $c^c(t, s, \delta)$, depends on the plant size as well as on the product durability and represents the equilibrium time path of a single firm. It has exactly the same form as the socially optimal solution in (9) where, however, the steady state c^* of the durable good and the root λ of the characteristic equation from (10) are replaced by

$$c^* := \frac{\alpha - (\rho + \delta)(M(\delta) - 2s + \tau_y) - \tau_w\delta - \tau_c}{2\lambda(\rho + \lambda)}, \quad \lambda := \frac{\sqrt{(\rho + 2\delta)^2 + 2\beta(n+1)} - \rho}{2}. \tag{23}$$

(23) is easily verified by applying the same procedure as in the case of welfare maximization (see appendix A1) to (1) and (22). The definition of λ again implies $\lambda > \delta$. Inserting the profit-maximizing production rate and the stock of the durable into the objective functional in (20) yields the maximum profit as a function of plant size and durability

$$\widetilde{\Pi}^c(s, \delta) = \frac{1}{\rho}\left(c^{*2}\lambda^2 z - s^2\right) \quad \text{with} \quad z := 1 - \frac{\beta(n-1)}{(\rho + \lambda)(\rho + 2\lambda)} \in \,]0, 1[.$$

z is less than one since $n > 1$ and greater than zero since $(\rho + \lambda)(\rho + 2\lambda) = \rho(\rho + \lambda) + 2\delta(\rho + \delta) + \beta(n+1) > \beta(n-1)$ owing to the definition of λ. From $\widetilde{\Pi}^c_s = 0$ we obtain

$$s^c = \frac{[\alpha - (\rho + \delta)(M(\delta) + \tau_y) - \tau_w\delta - \tau_c]\,(\rho + \delta)z}{2\,[(\rho + \lambda)^2 - (\rho + \delta)^2 z]} \tag{24}$$

as the profit-maximizing plant size. The second-order condition $\widetilde{\Pi}^c_{ss} < 0$ is satisfied since $\lambda > \delta$ and $z\epsilon]0,1[$. Using (24) we recalculate the steady-state value c^* from (23) as

$$c^* = \frac{[\alpha - (\rho+\delta)(M(\delta)+\tau_y) - \tau_w\delta - \tau_c]\,(\rho+\lambda)}{2\lambda\,[(\rho+\lambda)^2 - (\rho+\delta)^2 z]} \tag{25}$$

which allows us to express the maximum profit as a function of durability alone

$$\Pi^c(\delta) := \widetilde{\Pi}^c(s^c,\delta) = \frac{1}{4\rho}\,\frac{[\alpha - (\rho+\delta)(M(\delta)+\tau_y) - \tau_w\delta - \tau_c]^2 z}{(\rho+\lambda)^2 - (\rho+\delta)^2 z}. \tag{26}$$

In all cases investigated below, the various variables will be nonnegative due to assumption (14). The profit-maximizing durability is obtained by differentiating the maximum profit (26). For notational convenience, define $A(\delta) := \alpha - (\rho+\delta)(M(\delta)+\tau_y) - \tau_w\delta - \tau_c$ with $A'(\delta) = -[(\rho+\delta)M'(\delta) + M(\delta) + \tau_y + \tau_w]$, $A''(\delta) = -[2M'(\delta) + (\rho+\delta)M''(\delta)]$ and $B(\delta) := (\rho+\lambda)^2/z - (\rho+\delta)^2$. The maximum profit (26) is then written as $\Pi^c(\delta) = A^2(\delta)/4\rho B(\delta)$. The first and second-order conditions for a maximum of the function Π^c are $\Pi^{c\prime} = 0$ and $\Pi^{c\prime\prime} < 0$ or

$$-A'(\delta) + \frac{A(\delta)B'(\delta)}{2B(\delta)} = 0 \qquad \text{and} \qquad A''(\delta) < \frac{A(\delta)B''(\delta)}{2B(\delta)} - \frac{A'^2(\delta)}{A(\delta)}, \tag{27}$$

respectively. Thus, for the symmetric open-loop Nash equilibrium, equation (27) determines the profit-maximizing durability which depends on all model parameters and especially on the number of firms n. Again, for a fixed number of firms these conditions can only be investigated by numerical simulations. Before doing this, however, we will consider the case of an endogenous number of firms.

3.2 Free Entry - The Case of Perfect Competition

Saying that the number of firms is endogenous means that there is free entry to the market of the durable good. Under free entry, the market is entered by new firms so long as their expected profits are positive. Only when the expected profit becomes zero, no additional firm decides to produce the durable good. The following lemma describes the open-loop Nash equilibrium for this scenario.

Lemma 2 (Free entry) *The market equilibrium under free entry has the following properties:*

(i) The number of firms tends to infinity and the steady state is reached immediately. The maximum profit becomes zero for every firm.

(ii) The firm's plant size s^c, production rate $y^c(t)$ and stock $c^c(t)$ of the durable good converge to zero for all t.

(iii) The industry stock of the durable good becomes

$$C^c := \lim_{n\to\infty} nc^c(t) = \frac{\alpha - (\rho+\delta^c)(M(\delta^c)+\tau_y) - \tau_w\delta^c - \tau_c}{\beta} \qquad \text{for all } t. \tag{28}$$

(iv) The product durability $1/\delta^c$ satisfies the condition

$$(\rho+\delta^c)M'(\delta^c)+M(\delta^c)+\tau_w+\tau_y=0. \tag{29}$$

Proof: For any given and finite n, the maximum profit (26) is positive since B is positive due to $\lambda > \delta$ and $z\epsilon]0,1[$. For $n \to \infty$ the maximum profit becomes zero because $B \to \infty$. Thus, the number of firms under free entry tends to infinity. $n \to \infty$ implies $\lambda \to \infty$ owing to (23) and therefore $y^c(t) \to y^* = \delta c^*$ and $c^c(t) \to c^*$ owing to (9) in combination with (23). Hence, the steady state is reached immediately and *(i)* is completely proved. *(ii)* and *(iii)* are easily proved by computing the limit values of (24), (25) and nc^* for $n \to \infty$. To show *(iv)* let $n \to \infty$ in (27). A and its derivatives don't depend on n and therefore remain unchanged. However, $B'/2B \to 0$ and $B''/2B \to 0$ as $n \to \infty$. Hence, the first-order condition reduces to $-A' = 0$ or, equivalently, to (29). The second-order condition in (27) simplifies to $A'' < -A'^2/A$ the RHS of which is negative. Thus, as suggested in section 2, the second condition in (7) is *necessary* for the second-order condition to be satisfied. ■

The market share of every single firm in the free entry equilibrium tends to zero because the number of firms grows without bounds due to *(i)* of lemma 2. Regarding the demand function this means that the influence which the individual firm exerts on the market price disappears and every firm becomes a price-taker. Thus, perfect competition is reached. This interpretation of the free entry equilibrium is supported by two further results of lemma 2: First, the profit for every single firm becomes zero owing to *(i)*. Second, from *(iii)* and the linear demand function one obtains $M(\delta^c) + \tau_y = (P - \tau_w\delta^c - \tau_c)/(\rho + \delta^c)$. Hence, as expected for the market structure of perfect competition, the long-run unit cost (production cost plus tax payment) equals the capitalized net rental price (rental price less tax payments) of a unit of the output.[9]

We can now compare the equilibrium under perfect competition from lemma 2 with the social optimum from lemma 1. The firm's plant size, output rate and stock of the durable good under perfect competition converge towards their socially optimal values independent of the tax rates due to lemma 1 *(ii)* and lemma 2 *(ii)*. However, the evaluation of the product durability, the industry stock of the durable good and the amount of solid waste depends on the tax rates. Let us first consider the laissez-faire economy.

Proposition 2 *Suppose that all tax rates are zero. It then follows that the product durability under perfect competition is inefficiently low whereas the industry stock of the durable good and the amount of solid waste under perfect competition are inefficiently large.*

Proof: Define $F(\delta) := (\rho+\delta)M'(\delta) + M(\delta)$. Then the socially optimal durability is determined by $F(\delta^o) = -\text{MD} < 0$ owing to (19) and the profit-maximizing durability is

[9] SWAN (1970), p. 890 uses the same condition with zero tax rates as an *assumption* to derive the profit-maximizing durability under perfect competition in a laissez-faire economy. Regarding the relationship between the number of firms and the market structure of perfect competition our model is in line with the article of RUFFIN (1971) who establishes the same result for a static model of a nondurable good and the article of DOCKNER (1988) who establishes the same result for a dynamic model in which the price rather than the quantity of the good is captured by a differential equation.

determined by $F(\delta^c) = 0$ owing to (29) and $\tau_y = \tau_w = \tau_c = 0$. Since F increases with an increasing δ due to (7) it follows $\delta^c > \delta^o$ or, equivalently, $1/\delta^c < 1/\delta^o$. Hence, the product durability under perfect competition is inefficiently low. To prove the inefficiency of the industry stock, note that for zero tax rates, equation (28) becomes $C^c = (\alpha - (\rho + \delta^c)M(\delta^c))/\beta$. Compared with C^o from (18) we see that the denominator of C^c is lower than the denominator of C^o since $2\varepsilon(\delta^o)^2 > 0$ and that the numerator of C^c is greater than the numerator of C^o since $\gamma\delta^o > 0$ and $T(\delta^c) < T(\delta^o)$ with $T(\delta) := (\rho + \delta)M(\delta)$. To see the last statement, note that $T'(\delta) = F(\delta)$. Since F is an increasing function in δ due to (7) and since $F(\delta^c) = 0 > -\mathrm{MD} = F(\delta^o)$ this implies $T(\delta^c) < T(\delta^o)$ as suggested. In short we obtain $C^c > C^o$. Finally, the solid waste flow $\delta^c C^c$ under perfect competition is inefficiently large because both the decay rate and the industry stock are inefficiently large. ■

It is often argued that the product durability is socially optimal independent of the market structure since due to Swan's independence result it always equals the durability under perfect competition and this durability is efficient according to the first welfare theorem (Goering (1992), p. 58, Tirole (1994), p. 102). However, proposition 2 shows that such an interpretation is only permitted when the external environmental costs due to the solid waste of durable goods are completely ignored. Since in our model a decreasing durability leads to both a decrease in the production cost and a rise in the environmental cost, producers choose the durability so as to minimize their internal cost, thus ignoring the external damage of their production to the effect that the durability of their products is inefficiently low. Formally, this statement is captured by (29). For zero tax rates, the durability is profit-maximizing if the saved direct production costs $(\rho + \delta^c)M'(\delta^c)$ of an decreasing durability are just offset by the additional production costs $M(\delta^c)$ required to maintain the stock constant. In contrast to the social optimum, this condition ignores the external environmental damage, providing the producers the incentive to shorten the product durability. Moreover, the results of proposition 2 go beyond inefficient durability: The industry stock of the durable good and thus the solid waste flow under perfect competition are inefficiently large. Of course, this is an obvious result because the solid waste causes the environmental damage. However, the new insight of proposition 2 is that the inefficiently large quantity of waste is not only caused by an inefficiently large quantity of the consumption good but also by an inefficient product design, namely by an inefficiently low durability.

We turn to the case in which the policy maker uses taxes to influence the firm's behaviour in an effort to correct the environmental distortion.

Proposition 3 *The market equilibrium under perfect competition is socially optimal if and only if* $\tau_w + \tau_y = \mathrm{MD}$ *and* $\tau_c = \rho(\tau_w - \mathrm{MD})$.

The necessity of the proposed tax system is easily proved by setting $\delta^c = \delta^o$ in (28) and (29) and solving this for the tax rates with the help of (18) and (19). The sufficiency is shown by inserting the tax system into (18) and (19) and comparing this with (28) and (29). Proposition 3 states that the policy maker can achieve the social optimum simply by Pigouvian taxation, i.e. by setting the tax rate on solid waste equal to the marginal environmental damage and all other tax rates equal to zero. Separate instruments to correct the inefficient durability and the inefficient industry stock of the durable good are not required because both inefficiencies lead to an inefficiently large amount of solid waste and therefore can be corrected simultaneously by one instrument. In our framework the Pigouvian taxation is not restricted to a measure of taxation but it additionally captures two further policy instruments recently discussed in the solid waste management. First, the Pigouvian taxation can be interpreted as "eco-leasing" which means that the producers are required to rent their products and to pay the disposal costs after consumption (SOETE (1997)). In our model, producers are *assumed* to rent their output while τ_w may be interpreted as the costs which the producers pay directly for the disposal of a scrapped unit of the durable good rather than as a tax which they pay to the government. Second, the Pigouvian tax can be interpreted as a take-back requirement according to which the producers are indeed allowed to sell their output but are legally forced to remain the owner of the products, to take back the used units after consumption and to pay the disposal costs for the scrapped units (HOLM-MÜLLER (1997), chapter 5). The present model allows this interpretation, if we agree with SWAN's argumentation that renting the product is equivalent to selling the product (see p. 3). Of course, eco-leasing and take-back requirement are almost the same because both assign the property rights of the product over the *entire* product-life-cycle to the producers. Hence, eco-leasing and take-back requirement are special cases of what is called "extension of the producer responsibility" (OECD (1998)). In this sense, proposition 3 provides a contribution to the analysis of liability rules in environmental policy (for a brief survey see XEPAPADEAS (1997) pp. 69): The extended producer responsibility corrects the inefficiency in the product design as well as the inefficiency in the industry stock of the durable good and thus is able to internalize the external environmental costs of the solid waste flow.

In addition to the use of a single tax, proposition 3 suggests convex combinations of several tax rates capable of restoring the efficiency of the competitive solution. For example, if the policy maker is not allowed to tax the solid waste (due to political or legal constraints) then she can still achieve the social optimum by imposing a tax on the firm's output ($\tau_y = \text{MD}$) and a subsidy on the firm's stock of the durable ($\tau_c = -\rho\text{MD}$). The subsidy is needed because each unit of the good produced is taxed by a rate equal to the marginal environmental damage of the waste although not every unit of the good immediately turns into waste. A similar result holds, when the policy maker is restricted to waste taxation below the marginal environmental damage ($0 < \tau_w < \text{MD}$). Then she resolves efficiency as follows: The output must be taxed with the missing environmental damage ($\tau_y = \text{MD} - \tau_w > 0$) and a part of this additional tax must be reimbursed by subsidizing the firm's stock of the durable ($\tau_c = \rho(\tau_w - \text{MD}) < 0$). Again, this result has an interesting implication for the extension of the producer responsibility. If the extension is such that producers are responsible for only a part of the disposal process, then efficiency can only achieved by introducing further instruments. In Germany, for example, it is discussed whether the producer in the information technology industry should take back only a fixed percentage of their products and whether the costs of collecting the goods should be paid by the producers or by the local authorities (HOLM/MÜLLER (1997), p.

160f.). Our theoretical analysis shows that such regulations are only efficient if they are accompanied by further policy instruments.

3.3 Barriers To Entry - Numerical Results for Oligopoly

Now we turn to the case in which political constraints or prohibitively large entry costs restrict the emergence of new firms and thus the number of producers is fixed to a finite number n. The open-loop Nash equilibrium then describes an equilibrium for an oligopolistic industry in which the production rate, the firm's stock of the durable good, the plant size and the product durability are determined by the equations (9), (23) to (25) and (27). Since n is finite the steady state won't be reached immediately.

To find out whether this oligopoly is socially optimal, we have to compare the aforementioned conditions with the pertinent conditions from section 2. In general, this will be a sophistical work because the equations are much too complex. Thus, we restrict ourselves to some numerical simulations of the steady state. On the demand side assume $\alpha = 7500$ and $\beta = 150$ which implies a maximum demand of $\alpha/\beta = 50$. This may be interpreted as a prohibitive anual rental price of 7500 \$ per unit and a maximum demand of 50 mio. units. On the supply side, the long-run unit costs are specified by the function $M(\delta) = \kappa\delta^{\theta}$, where $\theta > 0$ denotes the elasticity of the long-run unit cost with respect to the product durability and $\kappa > 0$ denotes a scaling factor. Set $\theta = 0.9$ and $\kappa = 2500$ which implies that the long-run unit costs lie between 0 and 2500 \$ and increase less than proportionally with respect to the durability. Since we are not interested in convex combinations of the tax rates, we set all tax rates equal to zero except for the waste tax. Furthermore, let $\varepsilon = 0$. This is not a tight assumption because ε almost has the same effects as γ. Moreover, assume a discount rate of six percent per year, i.e. $\rho = 0.06$. The values for n, γ and τ_w are set alternatively according to the columns 1 to 3 of table 2. This table captures the numerical results for the product durability, the steady state industry stock of the durable and the steady state amount of waste in both the social optimum (columns 4 to 6) and the market equilibrium under oligopoly (columns 7 to 9).[10]

As a reference point, the rows 1 to 4 capture the case without environmental damage and without taxation. The product durability under oligopoly is then almost socially optimal (columns 4 and 7). This result is in line with GOERING (1992) who uses a two period model to show that the durability chosen by a renting oligopolist equals the durability under perfect competition which in the absence of externalities equals the socially optimal durability. The total industry stock of the durable under oligopoly turns out to be inefficiently small for finite n (columns 5 and 8). It becomes socially optimal when the number of firms becomes very large because then every firm becomes a price-taker (row 4, columns 5 and 8).

In the rows 5 to 8 a relatively large environmental damage of 1000 \$ per scrapped unit of the durable good is introduced. Compared with rows 1 to 4 we see that this leads to only a small decrease in the efficient industry stock of the durable (column 5) whereas the

---

[10] The results are obtained by solving (15) and (27) for the product durability. This was achieved with the help of a *regular falsi* algorithm. The program code can be obtained from the author upon request.

	Parameters			Social Optimum			Oligopoly		
column	1	2	3	4	5	6	7	8	9
row	n	γ	τ_w	δ^o	C^o	$\delta^o C^o$	δ^c	C^c	$\delta^c C^c$
1	2	0	0	0.5412	32.5956	17.6393	0.5431	21.7171	11.7947
2	10	0	0	0.5402	32.5897	17.6061	0.5567	29.6151	16.4865
3	1000	0	0	0.5400	32.5881	17.5977	0.5646	32.5539	18.3815
4	10^9	0	0	0.5400	32.5881	17.5976	0.5404	32.5881	17.6111
5	2	1000	0	0.2510	30.3457	7.6158	0.5431	21.7171	11.7947
6	10	1000	0	0.2509	30.3427	7.6131	0.5567	29.6151	16.4865
7	1000	1000	0	0.2509	30.3420	7.6124	0.5646	32.5539	18.3815
8	10^9	1000	0	0.2509	30.3420	7.6124	0.5404	32.5881	17.6111
9	2	100	0	0.4523	32.2671	14.5928	0.5431	21.7171	11.7947
10	10	100	0	0.4518	32.2621	14.5752	0.5567	29.6151	16.4865
11	1000	100	0	0.4517	32.2608	14.5707	0.5646	32.5539	18.3815
12	10^9	100	0	0.4517	32.2608	14.5707	0.5404	32.5881	17.6111
13	2	1000	1000	0.2510	30.3457	7.6158	0.2510	20.2268	5.0771
14	10	1000	1000	0.2509	30.3427	7.6131	0.2514	27.5816	6.9342
15	1000	1000	1000	0.2509	30.3420	7.6124	0.2516	30.3116	7.6270
16	10^9	1000	1000	0.2509	30.3420	7.6124	0.2509	30.3420	7.6128
17	2	100	100	0.4523	32.2671	14.5928	0.4530	21.5017	9.7410
18	48	100	100	0.4517	32.2610	14.5716	0.4613	31.6000	14.5759
19	1000	100	100	0.4517	32.2608	14.5707	0.4617	32.2279	14.8786
20	10^9	100	100	0.4517	32.2608	14.5707	0.4518	32.2608	14.5765

Table 2: Numerical results for the steady state of the oligopoly

efficient durability increases dramatically (column 4). The efficient amount of solid waste decreases (column 6) due to both the decrease in the industry stock and the increase in the product durability. Of course, the introduction of the environmental damage doesn't have an effect on the oligopoly solution at all (columns 7 to 9). This implies that the product durability under oligopoly is too small compared with the socially optimal one (columns 4 and 7). The oligopolistic stock of the durable good is inefficiently small for small n and ineffícently large for large n (columns 5 and 8). Alltogether, the amount of solid waste is greater under oligopoly than in the social optimum independent of the number of firms (columns 6 and 9). However, this is not true if we consider a relatively small external damage of 100 $ per unit (rows 9 to 12). The decrease in the socially optimal durability is then relatively small (column 4) and thus the amount of solid waste under oligopoly can even be inefficiently small (for example row 9, columns 6 and 9).

The effects of Pigouvian taxation are pictured in rows 13 to 16 for great damage and in rows 17 to 20 for small damage. For the computations in these rows, the tax rate on waste has been simply set equal to the marginal damage. For both large and small environmental damage, the Pigouvian tax provides an incentive for the oligopolists to choose the socially optimal durability (columns 4 and 7). In the case of large marginal damage, however, the industry stock of the durable and thus the amount of the solid waste under oligopoly is inefficiently *small* for finite n (rows 13 and 14, columns 5,6,8 and 9). This suggests that additionally a subsidy is required to correct the inefficiently small industry stock. The Pigouvian taxation ensures efficiency only if the number of firms becomes very large (row 16). Of course this is not surprising because for large n we reach the case of perfect competition from the previous section. The special feature of the case with small environmental damage is that Pigouvian taxation leads to an amount of solid waste which can be socially optimal even for a finite n near to 48 (row 18). However, this is a very special and highly random case.

In short we can conclude as follows. Analogously to the case of perfect competition (see proposition 2), the product durability in a laissez-faire equilibrium under oligopoly is inefficiently small when external environmental costs are explicitly taken into account. However, in contrast to the perfect competitive outcome, the amount of solid waste can be *smaller* than in the social optimum due to the inefficiently small industry stock. Pigouvian taxation (or the extension of the producer responsibility) indeed ensures an efficient product design under oligopoly but the amount of solid waste is inefficiently small in most cases due to an inefficiently small industry stock of the durable good. Hence, we would expect that efficiency requires an additional policy instrument like a subsidy on the oligopolist's stock of the durable.

4 Multi-Plant Monopoly

The numerical computations of the oligopoly case show that the Pigouvian taxation under imperfect competition is not sufficient for allocative efficiency because there is a second distortion with respect to the industry stock of the durable. However, it is somewhat unsatisfactory to obtain this result only on the basis of numerical simulations. Therefore, in this final section we will establish a similar result in the more general model for the extreme case of imperfect competition, namely for the monopoly case. We will also show that there is a crucial difference between monopoly and oligopoly.

One possibility to model the monopoly is to consider a firm with only one plant in the absence of any competitor ($n = 1$ in the analysis of section 3). As Kamien/Schwartz (1974) show, such a monopolist tends to design less durable consumption goods than firms under perfect competition. Swan (1977), however, argues that this model approach does not provide a sound basis to compare a monopoly with perfect competition because a *one-plant* monopoly is compared with a *multi-plant* competitive industry. He considers a monopoly with an infinte number of plants and shows durability to be the same under monopoly and perfect competition. We agree with Swan's argument and concentrate on the analysis of a multi-plant monopoly. The monopolist faces the problem of solving

$$\max_{y(t),s,\delta,n} \int_0^\infty e^{-\rho t}\Big(R(nc) - nK(y,s,\delta) - n\tau_y y - n(\tau_w \delta + \tau_c)\, c\Big)dt \tag{30}$$

subject to (1). This problem is very similar to the profit-maximization under competition in (20). However, the monopolist's revenue $R(nc) = ncP(nc)$ depends on the total stock of the durable rather than on the individual plant's stock. Thus, $R'(nc)$ equals $P(nc) + ncP'(nc)$ and $R'' < 0$ implies the revenue function to be concave in the total stock of the durable. Moreover, the monopolist has to pay taxes for every plant. This assumption is helpful when the monopoly solution is compared with the solution under perfect competition. Furthermore, for the sake of simplicity, we only consider the case of an endogenous number of plants.

According to the social planner the monopolist proceed step by step. First, she determines the time path of the production rate and the stock of the durable good for given values of the plant size, the durability and the number of plants. This problem is solved with the same optimal control methods already used for the solution of welfare maximization in section 2. Again the solution $y^m(t; s, \delta, n)$ and $c^m(t; s, \delta, n)$ has the same form as (9) but with different definitions of c^* and λ. Second, she inserts this solution into the objective functional in (30) to get the maximum profit as a function of the plant size, the durability and of the number of plants. From this function we easily obtain the profit-maximizing plant size

$$s^m = \frac{[\alpha - (\rho+\delta)(M(\delta)+\tau_y) - \tau_w\delta - \tau_c]\,(\rho+\delta)}{2\,[(\rho+\lambda)^2 - (\rho+\delta)^2]} \tag{31}$$

with $\lambda := \sqrt{(\rho+2\delta)^2 + 4\beta n} - \rho)/2 > \delta$. The steady state value for the plant's stock of the durable can be calculated as

$$c^* = \frac{[\alpha - (\rho+\delta)(M(\delta)+\tau_y) - \tau_w\delta - \tau_c]\,(\rho+\lambda)}{2\lambda\,[(\rho+\lambda)^2 - (\rho+\delta)^2]}. \tag{32}$$

The maximum profit as a function of the durability and the number of plants is $\Pi^m(\delta, n) = nA^2(\delta)/4\rho B(\delta, n)$ with $A(\delta) := \alpha - (\rho+\delta)(M(\delta)+\tau_y) - \tau_w\delta - \tau_c$ and $B(\delta, n) := (\rho+\lambda)^2 - (\rho+\delta)^2$. The first and second-order conditions for the profit-maximizing durability are $\Pi^m_\delta = 0$ and $\Pi^m_{\delta\delta} < 0$ or

$$-A'(\delta) + \frac{A(\delta)B_\delta(\delta,n)}{2B(\delta,n)} = 0 \qquad \text{and} \qquad A''(\delta) < \frac{A(\delta)B_{\delta\delta}(\delta,n)}{2B(\delta,n)} - \frac{A'^2(\delta)}{A(\delta)}, \tag{33}$$

respectively. The derivative of the maximum profit with respect to n is

$$\Pi^m_n(\delta, n) = \frac{A^2(\delta)}{4\rho B(\delta,n)}\left(1 - \frac{2\beta n(\rho+\lambda)}{(\rho+2\lambda)B(\delta,n)}\right). \tag{34}$$

Analogously to the social optimum, we can now establish the following lemma.

Lemma 3 (Monopoly) *If the numbers of plants is endogenous, then the monopoly solution has the following properties:*

(i) The number of plants tends to infinity and the steady state is reached immediately. The profit of the monopoly is greater than zero.

(ii) The individual plant size s^m, production rate $y^m(t)$ and stock $c^m(t)$ of the durable good

converge to zero for all t.

(iii) The total stock of the durable good becomes

$$C^m := \lim_{n\to\infty} nc^m(t) = \frac{\alpha - (\rho+\delta^m)(M(\delta^m)+\tau_y) - \tau_w\delta^m - \tau_c}{2\beta} \qquad \text{for all } t. \tag{35}$$

(iv) The product durability $1/\delta^m$ satisfies the condition

$$(\rho+\delta^m)M'(\delta^m) + M(\delta^m) + \tau_w + \tau_y = 0. \tag{36}$$

Proof: As Swan (1977), p. 233 shows in more detail, Π_n^m from (34) is positive for every finite n and zero for infinite n. Thus, the profit-maximizing number of plants is infinite which implies $\lambda \to \infty$, i.e. the steady state is reached immediately. The proof of *(i)* is completed by computing $\lim_{n\to\infty} \Pi^m(\delta^m, n) = A^2(\delta^m)/4\rho\delta^m > 0$. For $n \to \infty$, the proof of *(ii)* to *(iv)* is exactly the same as that for the free entry equilibrium in lemma 2. ■

Several findings of lemma 3 coincide with previous results from the literature: First, the unregulated monopolist captures a part of the consumer surplus, and thus her profits are positive. Second, as in the static textbook model of price theory, the monopolist's stock of the consumption good is half of the total stock under perfect competition due to the assumption of linear demand and quadratic production cost (compare (35) with (28)). Third, by comparing (36) with (29) we see that the product durability under monopoly equals the one under perfect competition. This is a generalization of Swan's independence result (for our framework the independence result is shown by Swan (1977)) to the case in which several tax rates are taken into account. Note, that the introduction of the environmental damage has no effect on the independence result since the damage influences only the socially optimal durability.

Due to the second and third findings we can conclude that the amount of solid waste under monopoly is half of that under perfect competition. However, the comparison of the monopoly with the social optimum is of greater interest. Consider first the case of a laissez-faire economy. The durability under monopoly equals the one under perfect competition according to Swan's independence result. Thus, zero tax rates imply the durability under monopoly to be smaller than in the social optimum owing to proposition 2. Moreover, by inserting the zero tax rates into (35) and comparing this with (18) we see that the difference between the monopolist's stock of the durable good and the social optimal one is not unique in sign. Thus we have proved the following proposition.

Proposition 4 *Suppose that all tax rates are zero. It then follows that the product durability under monopoly is inefficiently small whereas both the total stock of the durable good as well as the amount of solid waste under monopoly can be greater, equal or less than in the social optimum.*

The results of proposition 4 are analogous to the results for the oligopoly in the previous section: First, under laissez-faire the monopolist chooses an inefficiently small product durability because she doesn't internalize the external cost of the solid waste. Second, the stock of the durable good can be smaller or greater than the socially optimal one because the model exhibits two opposite effects on the stock of the durable good: The environmental externality tends to increase the stock beyond its efficient level whereas the market power of the monopoly tends to decrease the stock below its efficient level. Since this leads to an ambiguous relationship between the monopolist's stock of the durable and the socially optimal one and since the durability is different in both cases, we are not able to say anything about the efficiency of the solid waste flow under monopoly.

Now turn to the case in which the policy maker uses taxes to influence the behaviour of the monopolist. The following proposition is easily been proved by comparing (35) and (36) of lemma 3 with (18) and (19) of lemma 1.

Proposition 5 *The allocation under the multi-plant monopoloy is socially optimal if and only if* $\tau_w + \tau_y = \mathrm{MD}$ *and* $\tau_c = \rho(\tau_w - \mathrm{MD}) - \beta C^o$.

In contrast to the case of perfect competition (see proposition 3), proposition 5 shows that Pigouvian taxation alone does not lead to a socially optimal outcome under monopoly because $\tau_w = \mathrm{MD}$ implies $\tau_c = -\beta C^o < 0$. This result is intuitively clear because the policy maker has to correct two market imperfections, namely the environmental externality and the market power of the monopoly. Hence, she internalizes the external environmental damage by using the Pigouvian tax and subsidizes the stock of the durable good because once the externality is internalized the monopolist's supply is inefficiently small. This finding corresponds with previous contributions on the relationship between Pigouvian taxation and the market structure (for a survey see Xepapadeas (1997), chapter 5) in which it is shown that the optimal effluent tax under monopoly is smaller than the Pigouvian tax, i.e. smaller than the external environmental damage. Indeed, the waste tax in the above case equals the marginal damage, but the overall tax burden of the monopolist is lower than this marginal damage because she receives a subsidy for the stock of the durable. A similar interpretation of proposition 5 is possible when the policy maker is not allowed to tax waste at a rate equal to the marginal environmental damage. For example, if political constraints require the waste tax to be lower than the marginal damage ($\tau_w < \mathrm{MD}$), then the policy maker can achieve efficiency by an additional tax $\tau_y = \mathrm{MD} - \tau_w > 0$ on the output and a subsidy $\tau_c = \rho(\tau_w - \mathrm{MD}) - \beta C^o < 0$ on the stock of the durable. The subsidy again is greater than under perfect competition (see p. 17) due to the correction of two market imperfections being required.

It is interesting to note that the optimal tax schemes in the case of the monopoly are almost similar to those suggested by the numerical simulations of the oligopoly. The Pigouvian tax in both cases is capable of providing the incentive for an efficient product design, i.e. an efficient durability (for monopoly compare (36) for $\tau_w = \mathrm{MD}$ and $\tau_y = 0$ with (19), for the oligopoly see table 2 rows 13 to 20). Under both market structures, however, Pigouvian taxation generally doesn't lead to an efficient amount of solid waste since there is an additional distortion due to the stock of the durable. The notable difference between monopoly and oligopoly is that under the latter market structure there are exceptions to the just mentioned result, i.e. there are some special cases (row 18 of table 2) in which Pigouvian taxation ensures the overall efficiency of the oligopoly at least in the steady state. Such special cases are not possible under monopoly.

5 Conclusion

The proceeding analysis investigated a dynamic model of an industry which produces a durable consumption good. With respect to environmental economics, the distinctive feature of the model is that the product durability is endogenously determined by the producers in the process of product design. With respect to the literature on product durability, the distinctive feature of the model is that the scrapped units of the durable good turn into solid waste and cause external environmental damage. This environmental damage c.p. increases with an increase in the industry stock of the durable as well as with a decrease in the product durability. The principal insights of the model are:

(a) The socially optimal durability of the consumption good increases with an increasing marginal environmental damage, with an expanding market for durables and with a decreasing discount rate.

(b) In the laissez-faire equilibrium under perfect competition, the producers choose an inefficiently small product durability whereas the industry stock of the durable and the amount of solid waste are inefficiently large. This market failure can be corrected simply by Pigouvian taxation, i.e. by a tax on waste equal to the marginal environmental costs.

(c) In an unregulated equilibrium under imperfect competition, the durability is also too small compared with the socially optimal one while the industry stock of the durable good and the amount of solid waste, however, can be greater, equal or less than in the social optimum. Pigouvian taxation leads to an efficient durability but in general not to an efficient amount of solid waste because there is a second market imperfection due to the stock of the durable. For this imperfection an additional subsidy is required.

When the Pigouvian taxation is interpreted as an extension of the producer responsibility to the entire product-life-cycle, it turns out that the ability of this recently discussed instrument depends on the market structure. For every market structure the extension of the producers responsibility is capable of restoring the efficient product design, i.e. ensures an efficient product durability. However, under imperfect competition it is not sufficient to ensure overall efficiency. In this case further policy instruments are required.

Of course a limitation of the present paper is that the results of the oligopoly are restricted to numerical simulations. Especially for the analysis of the adjustment path a more general analysis is warranted. But to obtain useful results another framework like the two-period model already used in the literature is probably required because a dynamic game like the one used in the present paper is too complex to get useful results.

References

Benchekroun, H. and N. van Long (1998), Efficiency Inducing Taxation for Polluting Oligopolists, *Journal of Public Economics* 70, 325-42

Dinan, T.M. (1993), Economic Efficiency Effects of Alternative Policies for Reducing Waste Disposal, *Journal of Environmental Economics and Management* 25, 242-256

DOCKNER, E. (1988), On the Relation between Dynamic Oligopolistic Competition and Long-Run Competitive Equilibrium, *European Journal of Political Economy* 4, 47-64

EICHNER, T. and R. PETHIG (1999a), Embodied Material and Markets for Recycling, Waste Treatment and Disposal, *Discussion Paper No.* 73-99, University of Siegen

EICHNER, T. and R. PETHIG (1999b), Efficient Management of Recycling and Waste Disposal, *Discussion Paper No.* 76-99, University of Siegen

FULLERTON, D. and T.C. KINNAMAN (1995), Garabage, Recycling and Illicit Burning, *Journal of Environmental Economics and Management* 29, 78-91

FULLERTON, D. and W. WU (1998), Policies for Green Design, *Journal of Environmental Economics and Management* 36, 131-48

GOERING, G.E. (1992), Oligopolies and Product Durability, *International Journal of Industrial Organization* 10, 55-63

HIGHFILL, J. and M. MCASEY (1997), Municipal Waste Management: Recycling and Landfill Space Constraints, *Journal of Urban Economics* 41, 118-36

HOLM-MÜLLER, K. (1997), *Ökonomische Anreize in der deutschen Abfallwirtschaftspolitik*, Heidelberg: Springer

KAMIEN, M.I. and N. SCHWARTZ (1974), Product Durability under Monopoly and Competition, *Econometrica* 42, 289-301

KARP, L. and D. NEWBERY (1993), Intertemporal Consistency Issues in Depletable Resources, in : KNEESE, A.V. and J.L. SWEENEY (Eds.) *Handbook of Natural Resource and Energy Economics*, Vol. III, Amsterdam, Chapter 19

KOHN, R.E. (1995), Convex Combinations of Recycling Incentives, *Mathematical Computation Modeling* 21, 13-21

LAFRANCE, J.T. and L.D. BARNEY (1991), The Envelope Theorem in Dynamic Optimization, *Journal of Economic Dynamics and Control* 15, 355-85

MULLER, E. and Y.C. PERES (1990), Optimal Dynamic Durability, *Journal of Economic Dynamics and Control* 14, 709-19

OECD (1998), *Extended and Shared Producer Responsibility: Phase 2 - Framework Report*, Paris

PETIT, M.L. (1990), *Control Theory and Dynamic Games in Economic Policy Analysis*, Cambridge: Cambridge University Press

RUFFIN, R.J. (1971), Cournot Oligopoly and Competitive Behavior, *Review of Economic Studies* 38, 493-502

SCHMALENSEE, R. (1979), Market Structure, Durability and Quality: A Selective Survey, *Economic Inquiry* 17, 177-196

SIEPER, E. and P.L. SWAN (1973), Monopoly and Competition in the Market of Durable Goods, *Review of Economic Studies* 40, 333-51

SOETE, B. (1997), Ökoleasing - eine Innovation?, *Zeitschrift für angewandte Umweltforschung* 10, 66-76

SWAN, P.L. (1970), Durability of Consumption Goods, *American Economic Review* 60, 884-94

SWAN, P.L. (1977), Product Durability under Monopoly and Competition: Comment, *Econometrica* 45, 229-35

TIROLE, J. (1994), *The Theory of Industrial Organization*, Seventh Edition, Cambridge: MIT Press

TIETENBERG, T.H. (1988), *Environmental and Natural Resource Economics*, Second Edition, Glenview: Scott, Foresman and Company

WAGNER, T. (1992), Reichtum, Haltbarkeit und Abfall: Die Strategie der Dauerhaftigkeit, *Jahrbuch für Nationalökonomie und Statistik* 109, 501-22

XEPAPADEAS, A. (1997), *Advanced Principles in Environmental Policy*, Cheltemham: Edward Elgar

Appendix

A1. Solving the dynamic system (1) and (8)

Take the steady state as particular solution. From (1) and (8) the steady state values are

$$c^* = \frac{\alpha - (\rho+\delta)(M(\delta) - 2s) - \gamma\delta}{2\delta(\rho+\delta) + \beta n + 2\varepsilon\delta^2 n}, \qquad y^* = \delta c^* \tag{37}$$

The general solution to the inhomogeneous system (1) and (8) then reads $y^o(t; s, \delta, n) = a_1 e^{\lambda_1 t} + a_2 e^{\lambda_2 t} + y^*$ and $c^o(t; s, \delta, n) = a_3 e^{\lambda_1 t} + a_4 e^{\lambda_2 t} + c^*$ where $a_1, \ldots a_4$ are constants to be determined. λ_1 and λ_2 are roots of the characteristic equation $\lambda^2 - \rho\lambda - \delta(\rho+\delta) - (\beta n + 2\varepsilon\delta^2 n)/2 = 0$. The roots are of opposite sign, thus the steady state is a saddle point and we are allowed to concentrate on the stable part of the solution. Hence, take the negative root only and call it $-\lambda$ where λ is defined in (10). The general solution simplifies to

$$y^o(t; s, \delta, n) = a_1 e^{-\lambda t} + y^*, \qquad c^o(t; s, \delta, n) = a_3 e^{-\lambda t} + c^*. \tag{38}$$

To determine the constants a_1 and a_3, use the initial conditions $c(0) = 0$ and $\dot{c}(0) = y(0)$ to get $a_3 = -c^*$ and $a_1 = (\lambda - \delta)c^*$. Inserting this into (38) and rearranging proves equation (9) as the solution to the dynamical system (1) and (8). Note, that by rearranging (10) to $\rho + 2\lambda = \sqrt{(\rho+2\delta)^2 + 2\beta n + 4\varepsilon\delta^2 n}$, squaring and simplifying we get

$$2\lambda(\rho+\lambda) = 2\delta(\rho+\delta) + \beta n + 2\varepsilon\delta^2 n. \tag{39}$$

With the help of this equation the expression for c^* in (37) simplifies to (10).

A2. Comparative dynamic results for the social optimum

For notational simplicity denote the socially optimal values by δ and C instead of δ^o and C^o. Then totally differentiating (18) and (19) yields the matrix equation:

$$\begin{pmatrix} 1 & \dfrac{2\varepsilon\delta C}{\beta+2\varepsilon\delta^2} \\ 2\varepsilon\delta & 2M'+(\rho+\delta)M''+2\varepsilon C \end{pmatrix} \begin{pmatrix} dC \\ d\delta \end{pmatrix} = \begin{pmatrix} \dfrac{d\alpha - Cd\beta - Md\rho - \delta d\gamma - 2\delta^2 C d\varepsilon}{\beta+2\varepsilon\delta^2} \\ -M'd\rho - d\gamma - 2\delta C d\varepsilon \end{pmatrix}.$$

Denote the determinant of the first matrix by Δ. Then $\Delta = 2M'(\delta) + (\rho+\delta)M''(\delta) + 2\beta\varepsilon C/(\beta+2\varepsilon\delta^2) > 0$ due to assumption (7). The following comparative dynamic results are obtained by applying Cramer's rule:

$$\frac{\partial\delta}{\partial\alpha} = -\frac{1}{\Delta}\frac{2\varepsilon\delta}{\beta+2\varepsilon\delta^2} < 0, \qquad \frac{\partial C}{\partial\alpha} = \frac{1}{\Delta}\frac{2M'+(\rho+\delta)M''+2\varepsilon C}{\beta+2\varepsilon\delta^2} > 0,$$

$$\frac{\partial\delta}{\partial\beta} = \frac{1}{\Delta}\frac{2\varepsilon\delta C}{\beta+2\varepsilon\delta^2} > 0, \qquad \frac{\partial C}{\partial\beta} = -\frac{1}{\Delta}\frac{C[2M'+(\rho+\delta)M''+2\varepsilon C]}{\beta+2\varepsilon\delta^2} < 0,$$

$$\frac{\partial\delta}{\partial\rho} = \frac{1}{\Delta}\left(\frac{2M\varepsilon\delta}{\beta+2\varepsilon\delta^2} - M'\right) > 0, \qquad \frac{\partial C}{\partial\rho} = \frac{1}{\Delta}\frac{2M'\varepsilon\delta C - M[2M'+(\rho+\delta)M''+2\varepsilon C]}{\beta+2\varepsilon\delta^2} < 0,$$

$$\frac{\partial\delta}{\partial\gamma} = -\frac{1}{\Delta}\frac{\beta}{\beta+2\varepsilon\delta^2} < 0, \qquad \frac{\partial C}{\partial\gamma} = -\frac{1}{\Delta}\frac{[2M'+(\rho+\delta)M'']\delta}{\beta+2\varepsilon\delta^2} < 0,$$

$$\frac{\partial\delta}{\partial\varepsilon} = -\frac{1}{\Delta}\frac{2\beta\delta C}{\beta+2\varepsilon\delta^2} < 0, \qquad \frac{\partial C}{\partial\varepsilon} = -\frac{1}{\Delta}\frac{2\delta^2 C[2M'+(\rho+\delta)M'']}{\beta+2\varepsilon\delta^2} < 0.$$

The signs of these partial derivatives are summarized in the first two rows of table 1. The signs for the change in the amount of solid waste are easily obtained by $\partial(\delta C)/\partial k = C\cdot\partial\delta/\partial k + \delta\cdot\partial C/\partial k$, $k = \alpha, \beta, \rho, \gamma, \varepsilon$.

Computations not to be published

Proof of the Equation (4)

The second equation in (3) is an inhomogeneous linear differential equation in μ. The solution to the homogenous part $\dot{\mu} - \delta\mu = 0$ can be written as

$$\mu(t) = ae^{\delta t} \tag{40}$$

where a is an arbitrary constant. Now use the method of variation of the parameter. Let $a = a(t)$, differentiate (40) with respect to time and insert the result together with (40) into the inhomogeneous differential equation in (3). Solving the resulting expression with respect to $\dot{a}(t)$ yields

$$\dot{a}(t) = -ne^{-(\rho+\delta)t}\Big(S'(nc(t)) - \delta D'(n\delta c(t))\Big)$$

from which we obtain

$$a(t) = b - n\int_0^t e^{-(\rho+\delta)v}\Big(S'(nc(v)) - \delta D'(n\delta c(v))\Big)dv \tag{41}$$

with b as the constant of integration. Now insert (41) in (40) to obtain

$$\mu(t) = \Big[b - n\int_0^t e^{-(\rho+\delta)v}\Big(S'(nc(v)) - \delta D'(n\delta c(v))\Big)dv\Big]e^{\delta t}. \tag{42}$$

To determine b use the transversality condition $\lim_{t\to\infty}\mu(t) = 0$. Hence,

$$\lim_{t\to\infty}\mu(t) = 0 \qquad \Leftrightarrow \qquad b = n\int_0^\infty e^{-(\rho+\delta)v}\Big(S'(nc(v)) - \delta D'(n\delta c(v))\Big)dv.$$

Replacing b in (42) by this expression and simplifying yields equation (4).

Computation of the maximum welfare $\widetilde{V}(s, \delta, n)$

First, insert the consumer surplus as soon as the production cost and the environmental cost into the objective functional (2) to get the maximum welfare

$$\widetilde{V}(s,\delta,n) = n\int_0^\infty e^{-\rho t}\left((\alpha-\gamma\delta)\,c(t) - \frac{\beta n + 2\varepsilon\delta^2 n}{2}\,c^2(t) - y^2(t) - (M-2s)\,y(t) - s^2\right)dt$$

where $y(t)$ and $c(t)$ stand for the welfare-maximzing production rate and stock of the durable good from equation (9). By using these equations we can compute the single integrals as

$$\int_0^\infty e^{-\rho t}\,(\alpha-\gamma\delta)\,c(t)dt = c^*\lambda\,\frac{\alpha-\gamma\delta}{\rho(\rho+\lambda)},$$

$$\int_0^\infty e^{-\rho t}\,\frac{\beta n + 2\varepsilon\delta^2 n}{2}\,c^2(t)dt = c^{*2}\lambda^2\,\frac{\beta n + 2\varepsilon\delta^2 n}{\rho(\rho+\lambda)(\rho+2\lambda)},$$

$$\int_0^\infty e^{-\rho t}\, y^2(t) dt = c^{*2}\lambda^2\, \frac{2\delta(\rho+\delta)+\rho(\rho+\lambda)}{\rho(\rho+\lambda)(\rho+2\lambda)},$$
$$\int_0^\infty e^{-\rho t}\,(M-2s)\, y(t) dt = c^*\lambda\, \frac{(M-2s)(\rho+\delta)}{\rho(\rho+\lambda)},$$
$$\int_0^\infty e^{-\rho t}\, s^2 dt = \frac{s^2}{\rho}.$$

Now insert these expressions in $\widetilde{V}(s,\delta,n)$ and rearrange. The result is

$$\widetilde{V}(s,\delta,n) = \frac{n}{\rho}\Big(c^{*2}\lambda^2 z - s^2\Big) \tag{43}$$

with

$$\begin{aligned} z \;&:=\; \frac{\alpha-(\rho+\delta)(M-2s)-\gamma\delta}{c^*\lambda(\rho+\lambda)} - \frac{\rho(\rho+\lambda)+2\delta(\rho+\delta)+\beta n+2\varepsilon\delta^2 n}{(\rho+\lambda)(\rho+2\lambda)} \\ &\overset{(39)}{=} \; \frac{\alpha-(\rho+\delta)(M-2s)-\gamma\delta}{c^*\lambda(\rho+\lambda)} - 1 \overset{(10)}{=} 2-1=1. \end{aligned}$$

For $z=1$, equation (43) equals $\widetilde{V}(s,\delta,n)$ in the text.

Derivation of the equation (21)

If the plant size and the product durability are held constant, (20) becomes a problem of optimal control with the present-value Hamiltonian

$$\mathcal{H} = e^{-\rho t}\Big(R(c) - K(y,s,\delta) - \tau_y y - (\tau_w\delta+\tau_c)\, c\Big) + \mu\Big(y-\delta c\Big).$$

Because the term in the first brackets is concave in the production rate and the stock of the durable, the necessary and sufficient conditions for a maximum are

$$\mathcal{H}_y = -e^{-\rho t}\Big(K_y(y,s,\delta)+\tau_y\Big)+\mu = 0, \qquad \dot{\mu} = -e^{-\rho t}\Big(R'(c)-\tau_w\delta-\tau_c\Big)+\delta\mu. \tag{44}$$

Together with the transversality condition $\lim_{t\to\infty}\mu(t)=0$, the last of these equations can be solved by the same procedure as in the welfare maximization. The result is

$$\mu(t) = \int_t^\infty e^{-\rho v} e^{-\delta(v-t)}\Big(R'(c(v)) - \tau_w\delta-\tau_c\Big)dv. \tag{45}$$

$\mu(t)$ equals the net marginal value (marginal revenue less tax rates) of a marginal increase in the firm's stock of the durable good in period t and therefore may be interpreted as firm's shadow price of c. Combining (45) with (44) yields the marginal cost/marginal revenue condition for the production rate

$$K_y(y(t),s,\delta)+\tau_y+\int_t^\infty e^{-(\rho+\delta)(v-t)}(\tau_w\delta+\tau_c)dv = \int_t^\infty e^{-(\rho+\delta)(v-t)}R'(c(v))dv.$$

The total marginal cost (marginal production cost plus marginal tax payments) equals the marginal revenue of a marginal increase in output. Differentiating this condition with respect to time and rearranging yields (21).

Computation of the maximum profits $\widetilde{\Pi}^c(s,\delta)$

Insert the second-order approximation together with the equilibrium condition $c+\widetilde{C} = nc$ into the objective functional (20) to obtain the maximum profit of the individual firm

$$\widetilde{\Pi}^c(s,\delta) = \int_0^\infty e^{-\rho t}\Big((\alpha-\tau_w\delta-\tau_c)\,c(t) - \beta n\,c^2(t) - y^2(t) - (M-2s+\tau_y)\,y(t) - s^2\Big)dt.$$

For short, $y(t)$ and $c(t)$ stand for the solution of profit-maximization, $y^c(t;s,\delta)$ and $c^c(t;s,\delta)$, which is composed of (9) and (23). Use these equations to compute the single integrals as

$$\int_0^\infty e^{-\rho t}\,(\alpha-\tau_w\delta-\tau_c)\,c(t)dt = c^*\lambda\,\frac{\alpha-\tau_w\delta-\tau_c}{\rho(\rho+\lambda)},$$

$$\int_0^\infty e^{-\rho t}\,\beta n\,c^2(t)dt = c^{*2}\lambda^2\,\frac{2\beta n}{\rho(\rho+\lambda)(\rho+2\lambda)},$$

$$\int_0^\infty e^{-\rho t}\,y^2(t)dt = c^{*2}\lambda^2\,\frac{2\delta(\rho+\delta)+\rho(\rho+\lambda)}{\rho(\rho+\lambda)(\rho+2\lambda)},$$

$$\int_0^\infty e^{-\rho t}\,(M-2s+\tau_y)\,y(t)dt = c^*\lambda\,\frac{(M-2s+\tau_y)(\rho+\delta)}{\rho(\rho+\lambda)},$$

$$\int_0^\infty e^{-\rho t}\,s^2dt = \frac{s^2}{\rho}.$$

Inserting these expressions into $\widetilde{\Pi}^c(s,\delta)$ yields

$$\widetilde{\Pi}^c(s,\delta) = \frac{1}{\rho}\Big(c^{*2}\lambda^2 z - s^2\Big)$$

with

$$\begin{aligned} z \;&:=\; \frac{\alpha-(\rho+\delta)(M-2s+\tau_y)-\tau_w\delta-\tau_c}{c^*\lambda(\rho+\lambda)} - \frac{\rho(\rho+\lambda)+2\delta(\rho+\delta)+2\beta n}{(\rho+\lambda)(\rho+2\lambda)} \\ &\overset{(23)}{=}\; 2-\frac{\rho(\rho+\lambda)+2\delta(\rho+\delta)+2\beta n}{(\rho+\lambda)(\rho+2\lambda)} \overset{(23)}{=} 2-\frac{(\rho+\lambda)(\rho+2\lambda)+\beta(n-1)}{(\rho+\lambda)(\rho+2\lambda)} \\ &=\; 1-\frac{\beta(n-1)}{(\rho+\lambda)(\rho+2\lambda)}. \end{aligned}$$

where in the second row the relation $2\lambda(\rho+\lambda) = 2\delta(\rho+\delta)+\beta(n+1)$ due to the definition of λ in (23) has been employed.

Derivation of the equtions (31), (32) and $\Pi^m(\delta,n)$

For given plant size, durability and number of plants, (30) becomes a problem of optimal control with the present-value Hamiltonian

$$\mathcal{H} = e^{-\rho t}\Big(R(nc) - nK(y,s,\delta) - n\tau_y y - n(\tau_w\delta+\tau_c)\,c\Big) + \mu\Big(y-\delta c\Big)$$

Since the term in the first brackets is concave in the production rate and the stock of the durable, the necessary and sufficient conditions for a maximum are

$$\mathcal{H}_y = -ne^{-\rho t}\Big(K_y(y,s,\delta)+\tau_y\Big)+\mu = 0, \qquad \dot{\mu} = -ne^{-\rho t}\Big(R'(nc)-\tau_w\delta-\tau_c\Big)+\delta\mu. \quad (46)$$

Together with the transversality condition $\lim_{t\to\infty}\mu(t)=0$, the last of these equations can be solved by the same procedue as in the welfare maximization. The result is

$$\mu(t) = n\int_t^\infty e^{-\rho v}e^{-\delta(v-t)}\Big(R'(nc(v))-\tau_w\delta-\tau_c\Big)dv. \quad (47)$$

$\mu(t)$ may be interpreted as the monopolist's shadow price of c. Combining (47) with (46) yields the marginal cost/marginal revenue condition for production rate

$$K_y(y(t),s,\delta)+\tau_y+\int_t^\infty e^{-(\rho+\delta)(v-t)}\Big(\tau_w\delta+\tau_c\Big)dv = \int_t^\infty e^{-(\rho+\delta)(v-t)}R'(nc(v))dv.$$

Differentiating this condition with respect to time and rearranging yields

$$\dot{y} = \frac{(\rho+\delta)K_y(y,s,\delta)+(\rho+\delta)\tau_y-P(nc)-ncP'(nc)+\tau_w\delta+\tau_c}{K_{yy}(y,s,\delta)}. \quad (48)$$

Using the second-order approximation, this differential equation becomes

$$\dot{y} = (\rho+\delta)y+\beta nc-[\alpha-(\rho+\delta)(M(\delta)-2s+\tau_y)-\tau_w\delta-\tau_c]/2 \quad (49)$$

which together with (1) determines the profit-maximizing production rate and stock of the durable good. The solution to this system of differential equation, $y^m(t;s,\delta,n)$ and $c^m(t;s,\delta,n)$, equals exactly (9) where, however, c^* and λ are replaced by

$$c^* := \frac{\alpha-(\rho+\delta)(M(\delta)-2s+\tau_y)-\tau_w\delta-\tau_c}{2\lambda(\rho+\lambda)}, \quad \lambda := \frac{\sqrt{(\rho+2\delta)^2+4\beta n}-\rho}{2}. \quad (50)$$

Now insert the second-order approximation into the objective functional in (30) to get the maximum profit of the monopolist

$$\widetilde{\Pi}^m(s,\delta,n) = n\int_0^\infty e^{-\rho t}\Big((\alpha-\tau_w\delta-\tau_c)\,c(t)-\beta n\,c^2(t)-y^2(t)-(M-2s+\tau_y)\,y(t)-s^2\Big)dt.$$

Replace $y(t)$ and $c(t)$ by $y^m(t;s,\delta,n)$ and $c^m(t;s,\delta,n)$ and compute the single integrals as

$$\int_0^\infty e^{-\rho t}\,(\alpha-\tau_w\delta-\tau_c)\,c(t)dt = c^*\lambda\,\frac{\alpha-\tau_w\delta-\tau_c}{\rho(\rho+\lambda)},$$

$$\int_0^\infty e^{-\rho t}\,\beta n\,c^2(t)dt = c^{*2}\lambda^2\,\frac{2\beta n}{\rho(\rho+\lambda)(\rho+2\lambda)},$$

$$\int_0^\infty e^{-\rho t}\,y^2(t)dt = c^{*2}\lambda^2\,\frac{2\delta(\rho+\delta)+\rho(\rho+\lambda)}{\rho(\rho+\lambda)(\rho+2\lambda)},$$

$$\int_0^\infty e^{-\rho t}\,(M-2s+\tau_y)\,y(t)dt = c^*\lambda\,\frac{(M-2s+\tau_y)(\rho+\delta)}{\rho(\rho+\lambda)},$$

$$\int_0^\infty e^{-\rho t}\,s^2dt = \frac{s^2}{\rho}.$$

Inserting these expressions into $\widetilde{\Pi}^m(s,\delta,n)$ yields

$$\widetilde{\Pi}^m(s,\delta,n) = \frac{n}{\rho}\Big(c^{*2}\lambda^2 z - s^2\Big) \tag{51}$$

with

$$\begin{aligned} z &:= \frac{\alpha - (\rho+\delta)(M - 2s + \tau_y) - \tau_w\delta - \tau_c}{c^*\lambda(\rho+\lambda)} - \frac{\rho(\rho+\lambda) + 2\delta(\rho+\delta) + 2\beta n}{(\rho+\lambda)(\rho+2\lambda)} \\ &\overset{(50)}{=} 2 - \frac{\rho(\rho+\lambda) + 2\delta(\rho+\delta) + 2\beta n}{(\rho+\lambda)(\rho+2\lambda)} = 1 \end{aligned}$$

The last equality is obtained by $2\lambda(\rho+\lambda) = 2\delta(\rho+\delta) + 2\beta n$ due to the definition of λ in (50). The profit maximizing plant size (31) is obtained from $\widetilde{\Pi}^m_s = 0$, where the second-order condition $\widetilde{\Pi}^m_{ss} < 0$ is satiesfied due to $\lambda > \delta$. Inserting (31) in c^* from (50) and $\widetilde{\Pi}^m$ from (51) yields (32) and $\Pi^m(\delta,n)$ in the text.