Games and Information

MEPS Course

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Contents

Glorious days of information economics



George A. Akerlof, A. Michael Spence, Joseph E. Stiglitz,Nobel-prize winners 2001

Glorious days of contract theory


Oliver Hart, Bengt Holmström,Nobel-prize winners 2016

1 Background

The world is not always what it seems at first glance.

There are hidden secrets.

In this course you will learn to understand the difference between

Moral Hazard

Adverse Selection

Signaling

The Value of Information

1.1 Some Examples

- The aim of this course is to offer economic approaches to analyzing such problems.
- To achieve this goal, we start by examining a few examples.

Robinson Crusoe's problem to sell a goat

- Robinson Crusoe owns a goat yielding s liters of milk every day. He can imagine to sell the goat at a reasonable price.
- Friday thinks of buying the goat. He does not know whether the goat is milking well or not.
- $$U_R = \begin{cases} 2s, \text{ if he keeps the goat} \\ p, \text{ if he sells the goat} \end{cases}$$
- $$U_F = \begin{cases} 0, \text{ if does not buy the goat} \\ 3s - p, \text{ if he buys the goat} \end{cases}$$

- Friday knows that some goats milk well and others don't.
- He knows the distribution of the amount of milk s a goat usually yields. s is equally distributed on the interval $[0, 10]$.
- Friday is risk neutral. He uses the expected value of s^e as an estimator of the true yield of milk.
- A priori the estimator is $s^e = 5$.
- Robinson Crusoe is willing to offer the goat at a fair price p_{fair}.
- Friday concludes, that $p_{\text{fair}} \geq 2s$, in other words $0 \leq s \leq p_{\text{fair}}/2$.
- Hence Friday alines his estimator $s^e(p_{\text{fair}}) = p_{\text{fair}}/4$.
- He will accept Crusoe's offer, if U_F is non-negative, i.e. $3s^e(p_{\text{fair}}) \geq p_{\text{fair}}$.
- However, $3s^e(p_{\text{fair}}) = \frac{3}{4} p_{\text{fair}} < p_{\text{fair}}$!
- Whatever Crusoe may regard as a fair price, Friday will (mis-)interpret his offer and reject it.

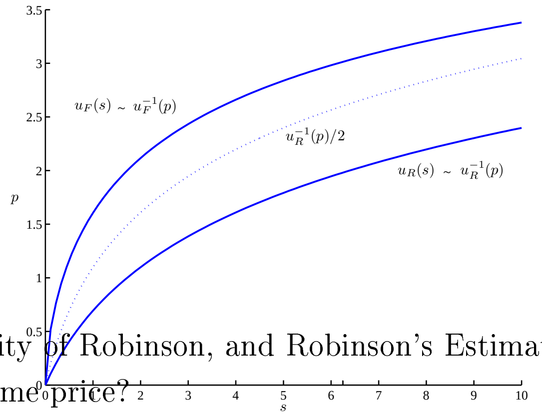
Does the result hinge on the parameters of the example?

One can set up a slightly more general model of the problem. For that, consider the following matrix of outcomes:

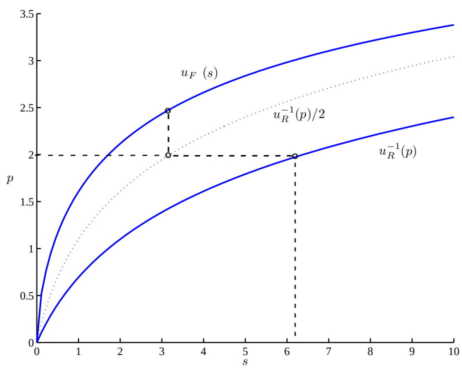
$$\text{Robinson's outcome} \quad \begin{cases} p & \textit{trade} \\ u_R(s) & \textit{no trade} \end{cases}$$

$$\text{Friday's outcome} \quad \begin{cases} u_F(s) - p & \textit{trade} \\ 0 & \textit{no trade} \end{cases}$$

- $u_R(s)$ and $u_F(s)$ are the respective values of milk for Robinson and Friday. $u_R^{-1}(p)$ and $u_F^{-1}(p)$ are the respective inverse valuation functions.
- Robinson knows the exact value s therefore also that of $u_R(s)$. But Friday has to use an estimate of s for valuation: $u_F(s^e)$.
- If Robinson is willing to sell his goat at price p_{fair}, then $p_{\text{fair}} \geq u_R(s)$, and Friday concludes $s \leq u_R^{-1}(p_{\text{fair}})$.
- Friday's estimate will be $s^e = u_R^{-1}(p_{\text{fair}})/2$.
- On the other hand Friday accepts p_{fair} if $u_F(s^e) \geq p_{\text{fair}}$.
- So p_{fair} has to satisfy the nested criterion $u_F(u_R^{-1}(p_{\text{fair}})/2) \geq p_{\text{fair}}$.
- Apply u_F^{-1} to both sides and get $u_R^{-1}(p_{\text{fair}})/2 \geq u_F^{-1}(p_{\text{fair}})$.
- In other words, Friday will accept a offer if the gain of trade outweighs the risk of overestimation of s.



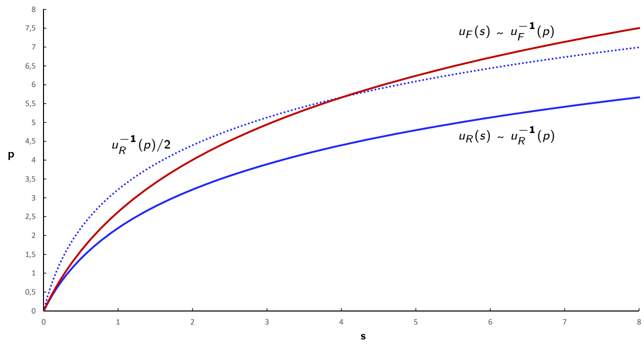
Utility of Friday, utility of Robinson, and Robinson's Estimator
Can they agree on some price?



Friday will accept any price because $u_R^{-1}(p)/2 \geq u_F^{-1}(p)$ for all p

The reasoning in detail:

- Robinson picks a price.
- Friday applies Robinson's inverse utility function to determine an upper bound for the quality.
- He then estimates the actual quality with on the basis of the quality distribution.
- He evaluates the estimated quality with his evaluation function.
- By comparing his valuation with the asking price, he comes to the conclusion that the purchase is worthwhile for him.



What is the conclusion for this example with a different pair of utility functions?

- If u_F is not much larger than u_R, the risk of buying a low quality spoils the deal.

Is this a real problem?
You may be aware of the fact, that a while ago large German car producers ran into problems because for years their management had been cheating on the declaration of pollutant emission of their diesel vehicles. The public discussion of the problem caused a (slight) disarrangement of car markets world wide.

A car market obstacle
In a radio broadcast I heard the following statement:

Obviously demand will decrease as prices go down!

One may be puzzled because usually we expect demand to decrease with prices.

- The broadcast statement is suggesting the opposite
- However, here the question arises what the true value of a car may be?
- Even if the producer of the car may have to pay for part of the extra costs to solve the problem with the car, the resale value will probably decline.
- Current prices for new cars of this brand are likely to decrease because buyers are expecting inconveniences.

- The lower the current price of the car the more the potential buyers will expect the resale price to drop.
- Even worse: Potential buyers may be afraid that the current seller wants to get rid of the car because he is afraid the problem may be more severe than admitted by the seller.
- Hence, at least some buyers may refrain from buying this brand. And its is unlikely that you find new potential buyers.
- This phenomenon is called *adverse selection*.
- Can you build a little model to check whether the argument goes through?

Auctions
Now, Morteza wants to buy a very unique mug with the Eiffel Tower painted on it from a special souvenir shop. The shop owner is aware of his position as a monopolist, but has no idea about Morteza's willingness to pay and that of other potential clients. The number of interested customers is not very large, but too large to negotiate with each customer individually.

Therefore, he decides to auction off this special mug.

There are many forms of auctions. Here is the short list of the basic ones

- English Auction: Bidding starts at a low price and is raised incrementally as progressively higher bids are solicited, until no higher bids are received.

- Dutch Auction: Bidding starts with a high price and is decreased step by step by the auctioneer until the bid is accepted by a buyer.

What form of auction should the seller choose in order to maximize his revenue?

Assume that each bidder *privately* and *independently* forms an opinion of the value of the mug. Consider at first the case of an English Auction.

- Morteza and other bidders continue to participate until the price reaches their own private values.
- The auction stops when the bidder with the second-highest value drops out.
- Therefore, the seller's expected price is the expected value of the second-highest private value (plus the marginal increment of the last bidder).

Now consider the Dutch Auction.

- Morteza and his competitors plan to call out when the price has fallen slightly below their private valuations.
- The seller's expected price is the expected value of the highest private value minus the incremental amount by which this bidder allows the price to drop below his private value.
- Again, the seller's expected price turns out to be close to the expected the second-highest private value.

Which form of auctioning should the shop owner choose?

- In the case of an independent private value model, it doesn't matter!
- The result is known as the *revenue equivalence theorem*[1].

[1] A formal statement and proof needs a more elaborate specification of the framework.